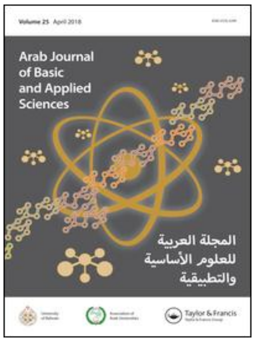


Arab Journal of Basic and Applied Sciences

ISSN: (Print) 2576-5299 (Online) Journal homepage: http://www.tandfonline.com/loi/tabs20

Public perception of algal consumption as an alternative food in the Kingdom of Bahrain

Salwa Al-Thawadi

To cite this article: Salwa Al-Thawadi (2018) Public perception of algal consumption as an alternative food in the Kingdom of Bahrain, Arab Journal of Basic and Applied Sciences, 25:1, 1-12, DOI: 10.1080/25765299.2018.1449344

To link to this article: https://doi.org/10.1080/25765299.2018.1449344

Published online: 18 Apr 2018.

ARAB JOURNAL OF BASIC AND APPLIED SCIENCES
2018, VOL. 25, NO. 1, 1–12
https://doi.org/10.1080/25765299.2018.1449344

University of Bahrain
Taylor & Francis
Taylor & Francis Group

ORIGINAL ARTICLE

OPEN ACCESS Check for updates

Public perception of algal consumption as an alternative food in the Kingdom of Bahrain

Salwa Al-Thawadi

Department of Biology, University of Bahrain, Sukhir, Kingdom of Bahrain

ABSTRACT

Algae are living organisms with high nutritional benefits. As such, algae are considered a solution to malnutrition and starvation. Individuals in the Gulf Corporation Council (GCC) region have limited food resources and face problems linked to malnutrition. Therefore, the introduction of a new food to their diet, such as algae, would be beneficial. However, these populations have conservative food habits and might not accept such anew food. Therefore, here we assessed consumer acceptance of natural and processed algae (seaweeds and *Spirulina*) in the Kingdom of Bahrain using a Technology Acceptance Model (TAM) approach. TAM is normally used to study the acceptance of technology, including commercial, industrial and nutritional industries. Here, we investigate the Bahraini community's likelihood of accepting algal food as an alternative food source. In addition, factors impacting the acceptance of algal food as an alternative food were examined. Valid questionnaires (300) were collected to empirically test the research model using the partial least square (PLS) path modelling approach. We found that the following proposed hypotheses were supported, except for the relationship between perceived healthiness of food and behavioural intention. This study revealed that sensory aspects, perceive healthiness of food, and knowledge experience/familiarity have a significant positive direct relationship to perceived risk and uncertainty while having an indirect relationship with behavioural intention to consume the algal product. Subjective norm, perceived risk and uncertainty, food neo-phobia, and consumer decision to eat algal food products were found to directly influence consumers' algal food behavioural intention, which, in turn, affects the consumers' decisions about whether to consume algal food products. Our data suggest that the people in the Kingdom of Bahrain are willing to consume algae and, thus, that the Bahraini market is ready to receive algal food products.

ARTICLE HISTORY
Received 24 March 2017
Accepted 24 October 2017

KEYWORDS
Algae; Seaweeds; Technology Accepted Model; perceived risk and uncertainty; behavioural intention

1. Introduction

1.1. Algae as an alternative food

Algae are a diverse group of organisms found in multiple environments, especially seawaters and ponds. Algae use sunlight to photosynthesize. Algae can be classed as either macroalgae (seaweeds, large in size, over 150 feet long) or microalgae (that cannot be seen by the naked eye). Many species of macro- and microalgae are used for food, food additives, animal feed, fertilizers and biochemicals (Henrikson, 2009; Thomas, 2002). Algae are considered a main food and medicine source for Asian people. The nutritional benefits of algae make them a possible solution to malnutrition and starvation. In many centuries seaweeds are used as a traditional food. This custom has been spread by people from China, Japan, and the Republic of Korea as they migrated around the world so that today there are many countries in which seaweed is consumed (FAO, 2003). Most seaweeds are rich in protein, polysaccharides, minerals, antioxidants and significant amount of lipid and vitamins (Stévant, Rebours, & Chapman, 2017). They can be a source of essential fatty acids that reduce the risk of heart disease (Khotimchenko & Kulikova 2000; Maehre, Malde, Eilertsen, & Elvevoll, 2014; Sańchez-Machado et al., 2004). Because of its health benefits and nutritive value, consuming algae is recommended as a means of enriching the diet and preventing diseases, such as cancer, cardiovascular disease and cerebrovascular disease, as well as iodine deficiency (Cordain, Eaton, Miller, Mann, & Hill, 2002). Therefore, certain countries, especially those with malnutrition problems or that do not already depend on algae in their diet, are urged to adopt algae as food. Therefore, in 1992, the USDA suggested that seaweeds (considered a well-balanced, healthy sea-vegetable) should be allocated to the bottom area of the food pyramid (together with typical vegetables and fruit) (USDA, 1992).

CONTACT Salwa Al-Thawadi salwaalthawadi@gmail.com, salthawadi@uob.edu.bh Department of Biology, University of Bahrain, Sukhir, Kingdom of Bahrain

1.2. Algae as a potential source of superfood in the Middle East

Recently, edible seaweed products have become popular in the food industry in several countries (especially Japan) because of their interesting medicinal properties (Kılınç et al., 2013). In the Middle East, there has been a trend to enrich the diet using algae. For example, in Jordan, children at public schools are fed *Spirulina (Arthrospira platensis)*. *Spirulina* is a microalga composed of nutrients like protein, mineral salts (calcium, phosphorus, magnesium, zinc, copper, iron, chromium, manganese, sodium, potassium, and selenium); enzymes; antioxidants; vitamins (beta carotene, vitamin A, vitamins B1, B2, B6, B12, C and E) essential amino acids; and rare essential lipids such as gamma linolenic acid (Gutiérrez-Salmeán, Fabila-Castillo, & Chamorro-Cevallos, 2015; Ismail & Hong, 2002; Kent, Welladsen, Mangott, & Li, 2015; Matondo, Takaisi, Nkuadiolandu, Lukusa, & Aloni, 2016). A study in Lebanon identified microalgae species that have the potential for use as a superfood or as cheap renewable energy (Makki, 2014). Moreover, in the Middle East and other poor countries, the Intergovernmental Institution for the use of Microalgae *Spirulina* Against Malnutrition (IIMSAM) supports children and adults suffering from malnutrition and other disorders resulting from undernourishment (IIMSAM, 2008).

In the GCC region, seaweeds have been used as fish bait by threading algae along the hook. Old fishermen were occasionally consuming the seaweeds during bait preparation. However, it is not well known that people in this region use algae as a source of food. Seaweeds are used for purposes other than feeding people. For example, seaweeds are used as decoration and for adding nutritional value to chicken feed (El-Deek et al., 2011). Previously, UAE grew *Spirulina* in the new islands for decoration. More recently, the government in UAE have developed a strategy to become the leader in the cultivation of *Spirulina* against malnutrition, establishing a *Spirulina* farm, which is the first of its kind in the Middle East (UAEinteract, posted on 26/7/2009). In Saudi Arabia, the use of algae as a food source is under research. For example, the extracts of three algal samples from Chlorophyta (*Ulva lactuca*), Phaeophyta (*Sargassumcrassifolia*) and Rhodophyta *(Digeneasimplex*) were chemically analysed. These three algal extracts had different antioxidant activities, as well as different profiles of sugars, uronic acids, amino acids and small amounts of betaines (Al-Amoudi, 2009). Another Saudi Arabian study investigated the use of brown algae in chicken feed. The use of algae resulted in an increase in chicken body weight, egg production and egg quality. Furthermore, there was an increase in the immunity of the birds, as well as in their antioxidant and selenium content. The findings of these Saudi Arabian studies might encourage the people in the Gulf region to use these algae as a superfood (Al-Harthi & Al-Deek, 2011), especially in light of the concern about meeting basic food needs in the GCC region.

Novel food products are being developed at an increasing rate. Unfortunately, how people react to these products is understudied (Tenbült, de Vries, Dreezens, & Martijn, 2008). Although novel food products are continually introduced into the markets, their failure rate has been estimated as 60% (Costa & Jongen, 2006; Grunert & Valli, 2001) with few products surviving in the long term. Consumer acceptance is a major factor in the success of a novel food product. Thus, consumer perception of food should be studied in order to identify critical factors that lead to their acceptance or rejection.

The people of the Arabian Gulf suffer from a malnutrition problem due to an unhealthy nutrition pattern (Musaiger, Hassan, & Obeid, 2011); therefore, enhancing the diet of people in this region is important. Owing to their nutritional value, we propose that algae should be introduced to the region as a high nutritional source of food. However, it is difficult to change the food habits of these peoples and to introduce new foods. Here, we develop a framework for consumer acceptance of algal products in the Kingdom of Bahrain using a Technology Acceptance Model (TAM) approach. In addition, factors impacting the acceptance or rejection of algal food as an alternative food are studied. Those factors investigated here include socio-demographic determinants, cognitive and attitudinal determinants (knowledge and experience/familiarity), health consideration, the perception of risk and uncertainty, sensory appeal, subjective norm and food neo-phobia (Prescott, Young, O'Neill, & Yau, 2002; Ronteltap, Trijp, Renses, & Frewer, 2007; Verbeke, 2005; Vidigal et al., 2015).

1.3. Research contribution

There is not much known about consumer perception of algal food products in the Middle East particularly in the Kingdom of Bahrain. To the best of our knowledge, this study is the first in the Middle East to investigate customer acceptance towards using algae as a food source. Due to the social, economic situation, people in the Middle East face a serious problem in nutrition-related diseases, such as growth retardation among young children and micronutrient deficiencies due to inadequate consumption of nutrients; in addition to those diseases which are associated with altering life style such as cardiovascular disease, cancer, osteoporosis, diabetes and obesity (Musaiger et al., 2011). The Arab communities need to be introduced to novel food with a high nutritional value such as algae. Moreover, algae are needed for sustainable development and food security as projects of algal cultivation in the region. For this purpose, the perception of eating algae in the region is necessary to be studied.

2. Research model and hypotheses

The main objective of this study is to investigate those factors that influence the acceptance of algae (Seaweeds/*Spirulina*) as an alternative food by the community of the Kingdom of Bahrain. The literature exhibited many factors that might affect the decision to accept and consume algae as an alternative food in any country (Prescott et al., 2002; Ronteltap et al., 2007; Verbeke, 2005).

In the current study, four factors have been selected that might direct effect on behavioural intention to accept algal as a food: (1) socio-demographic factors (education, gender, body weight, diseases); (2) subjective norm; (3) perceived risk and certainty; and (4) food neo-phobia. Moreover, factors such as innovative features sensory appeal, perceived healthiness of the food and knowledge experience/familiarity have been selected as potentially an indirect effect on behavioural intention to accept consuming algae. The behavioural intention, however, is proposed as having a direct effect on the customers' decisions about accepting and consuming algae (Seaweeds/*Spirulina*). As discussed earlier, concerning consumer acceptance of food technologies in previous work, different elements affecting the consumer acceptance which will be integrated into the conceptual framework (model) were assessed. The research model is depicted in Figure 1.

2.1.1. Sensory aspect

Sensory factors including appearance, colour, texture, taste and smell are important for the evaluation of food products, affecting perception and food acceptance (Grunert, Bredahl, & Scholderer, 2003; Ng'ong'ola-Manani, Mwangwela, Schüller, Østlie, & Wicklund, 2014). Sensory quality is necessary for the success of a product (Grunert, 2005). Consumers' intention to buy genetically modified cheese was improved by its sensory qualities (Lahteenmaki et al., 2002). Moreover, the appearance of food increases purchasing attitude. Another study by Radder and Roux (2005) that assessed the quality of food according to South African customers found that more than half of the respondents considered colour and smell as key indicators for determining the quality of meat, followed by texture. South African costumers identified flavour, tenderness and juiciness as the main indicators of the taste of red meat. Thus, sensory aspects related to food can affect the levels of uncertainty about a food product. Generally, people with lower uncertainty avoidance have a tendency to be more tolerant of risk (Jacqueline & Julie, 2002). Therefore, the following hypothesis was developed:

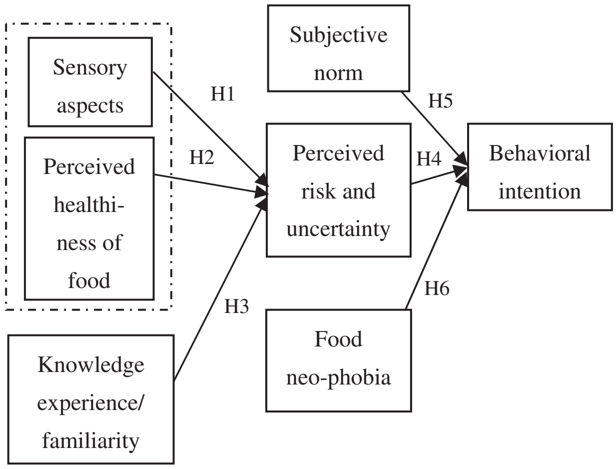


Figure 1. Conceptual framework determinants of attitude towards using algae as alternative food source.

H1: Sensory appeal has a positive impact on perceived risk and uncertainty.

2.1.2. Perceived healthiness of food

Health is important for mankind; some consumers choose food products that they believe will help to maintain their health and meet their nutritional needs (Pollard, Kirk, & Cade, 2002). High nutritional value and low-calorie content are the main concerns for many customers when choosing food. Algae often have high contents of polyunsaturated fatty acids, which play a critical role in human metabolism (Koller, Salernob, & Braunegg, 2015). Moreover, algae are used to cure diseases such as goitre, intestinal afflictions, cancer, cervix dilation, Alzheimer's, constipation, urinary tract infections, diarrhoea, breast infections, tuberculosis, headaches, scabies, cardiovascular disease and fungal infections, as well as for cholesterol reduction, bleeding control, vermifuge, breaking of fevers and as a wound dressing (Levine, 2016; Olasehinde, Olaniran, & Okoh, 2017). Owing to their health benefits, algae can be easily accepted as a food product. Therefore, the following hypothesis was developed:

H2: Perceived healthiness of algae (food) has a positive impact on perceived risk and uncertainty.

2.1.3. Knowledge and experience/familiarity

In general, there is a complex relationship between knowledge, expertise and risk. Consumer preferences for continuous innovations increase when they have related knowledge (Ronteltap et al., 2007). Uncertainty exists when details of situations are ambiguous or unpredictable; when information is unavailable or inconsistent; and when people feel insecure about their own knowledge or the state of knowledge in general (Ronteltap et al., 2007). In genetically modified food (GMF), knowledge and expertise were always found to be determinants of risk perceptions and attitude towards accepting it. Consumers' risk perception of a broad range of hazards is increased by expertise (Bouyer, Bagdassarian, Chaabanne, & Mullet, 2001), whereas attitude is positively affected by available knowledge about gene technology and GMF (Siegrist, 1998; Verdurme & Viaene, 2001). In a study related to irradiated beef perception, knowledge about food safety appears to reinforce consumer intention to purchase irradiated beef (Rimal, McWatters, Hashim, & Fletcher, 2004). The role of knowledge in the acceptance of novel technologies was supported by another study, which stated that a lack of experience, knowledge and know-how could negatively affect the attitude towards accepting food (Radder & Roux 2005). On the other hand, the familiarity of new food will enhance its acceptance. This statement is supported by the spread of the use of algae as a food source in California and Hawaii, because of the large size of their Japanese communities (FAO, 2003). These communities spread the use of seaweeds through supermarkets and restaurants so that the taste of algae became familiar to the public. As a consequence, some companies have begun cultivating seaweeds onshore and in tanks, specifically for human consumption. The markets available to these companies are growing in California and Hawaii, and some companies are now exporting to Japan. Therefore, the following hypothesis was developed:

H3: Knowledge and experience have a positive impact on perceived risk and uncertainty.

2.1.4. Perceived risk and uncertainty

The emerging technologies in the food industry possess many risk characteristics, thus having a perceived threat to costumers' health. These technologies include genetic modification, food enhanced by nanotechnology, additives, preservatives and packaging technology. To utilize the food product, the consumer should be convinced that his/her life is safe and will not be affected adversely by eating a novel food.

The most critical factor influencing selecting and using a product is the potential risk associated with the product, as the consumers are concerned about the harm and unknown health risks caused by the new technologies (Cardello, 2003; Cardello, Schutz, & Lesher, 2007). The quality of food is invisible and uncertain, so it is difficult to be estimated by the consumers (Cardello, 2003). Others consumers are open to innovation and trust the new food technologies and their ability to offer new benefits (Bruhn, 2007). Those believers in the new food technologies accept novel food processing technologies easily (Bord & O'Connor, 1990). Therefore, the following hypothesis was developed:

H4: Perceived risk and uncertainty has a positive impact on behavioural intention.

2.1.5. Subjective norm

The subjective norm is defined as the perceived social influences to like or dislike a particular behaviour. The role played by subjective norms in purchasing behaviour was confirmed by Pande and Soodan (2015). Consumers are influenced to consume new food by trusted people, leaders' opinions, and relevant prestigious organizations (Bruhn, 2007). However, perceived behavioural control and social norm have received less attention or no attention in the literature on consumer acceptance of food technology and innovations. On the contrary, Choo, Chung, and Pysarchik (2004) stated that subjective norm was vital to consumers attitude towards new food technologies. Thus, subjective norm towards consuming algae might affect the behavioural intention. Therefore, the following hypothesis was developed:

H5: Subjective norm has a positive impact on behavioural intention.

2.1.6. Food neo-phobia

Food neo-phobia is normally related to the reluctance to eat new foods in which the consumer rejects consuming the unfamiliar food (Loewen & Pliner, 2000). Particular attitudinal, personality and lifestyle characteristics might also impact consumers' attitudes towards acceptance or rejection of new technology. The presentation of a new food product probably initiates a fear response within the individual (Barcellos et al., 2010). Pliner and Hobden (1992) developed the Food Neo-phobia Scale to assess individuals' willingness to try novel foods. The Food Neo-phobia Scale can identify those individuals who reveal a strong aversion to unfamiliar foods, who are referred to as neo-phobics. A lower acceptance of foods among consumers with a high level of food neo-phobia was reported (Labrecque, Doyon, Bellavance, & Kolodinsky, 2006). Other researchers found considerable differences in consumer's acceptance of unfamiliar food (beverages and fruits) between the low and high level of food neo-phobics (El Dine and Olabi, 2009; Sabbe, Verbeke, Deliza, Matta, & Van Damme, 2009). Therefore, the following hypothesis was developed:

H6: Food neo-phobia has a positive impact on behavioural intention.

3. Research methodology

To achieve the objectives of the current research, both experimental and survey approaches were used. An experiment was conducted to examine the behaviour of 30 people from University of Bahrain (UOB, students, and staff) in an algal taste test. This experiment is a pilot study to test the preliminary behaviours of UOB staff and students towards eating seaweeds. The outcome of the observations was reflected in the survey and in a second follow-up experiment. Participants were asked to taste seven types of algae (different types of pure natural seaweeds, crispy seaweeds, spicy seaweeds, biscuits supplemented with seaweeds and *Spirulina*), orange juice with seaweeds, and mango juice with seaweeds. The type of seaweeds was Nori (*Porphyra sp.*), red dulse (*Palmaria sp.*), Wakame (*Undaria pinnatifida*) and Hijiki (*Hijikia fusiformis*). Another experiment was done in Bahrain Garden Show (BIGS-2012), where almost 1000 individuals from different sectors and social ranks (including ministers, parliament members, Shurat Council members and the public) of the Kingdom of Bahrain were asked to taste algae. Different types of natural and processed seaweeds, *Spirulina*, different types of cheese with an Arabic taste but supplemented with seaweeds, and bread baked with a mixture of dried seaweeds were shown in the pavilion. The visitors were first introduced to general information about algae and their remarkable benefits, thereby encouraging them to taste algal food. A reasonable acceptance for algae consumption was noticed, especially after clearing the misconceptions around algae. Many of the participants returned to the pavilion to re-taste the different types/mixtures of algae and food supplemented with seaweeds. Due to the success of the two experiments, it was decided to measure the acceptance of people in the Kingdom of Bahrain of including algae in their diet. For this, a questionnaire was developed to measure the factors that determine acceptance of seaweeds consumption. Five-hundred questionnaires were distributed among the different segments of the Bahraini community. Of these, 300 were returned; a response rate of 60%, which is considered as a high response rate and is acceptable in such studies.

University of Bahrain students participated in the first part of the current study. The involvement of those students was due to the belief in the critical role imposed by higher education in the Arabian Gulf in measuring the scientific awareness among students. As examples for measuring the awareness among GCC University students, Universities in the United Arab Emirates (UAEU) and Kingdom of Bahrain (UOB) measured the student' awareness of biotechnology and global warming. An overall weak performance of UAEU students' understanding towards biotechnology was recorded (AbuQamar et al., 2015). This performance indicated the importance of enhancing students' technology awareness via education. The positive role of educational, academic curriculum in scientific awareness was further confirmed in another study at UOB (Freije et al., 2017).

4. Data analysis and results

4.1. Experiments

The following section presents the results of an experiment involving 30 UOB participants, of which 20 were Bahrainis (Table 1). The demographic results of the responders show that the participants were students ($n = 14$), administrative ($n = 10$), academics ($n = 5$) or other ($n = 1$) (Table 1). Fourteen of the participants were 16–20 years old, 6 participants were 20–40 years old and 10 participants were 40–60 years old (Table 1). The majority of the participants were males ($n = 18$).

The results revealed that most of the participants accepted to taste the seaweeds in one or more of the exhibited algal types (naturally dried and processed) with different tastes. Thus, around 80% of the participants accepted to taste the orange/mango juice with seaweeds and seaweeds biscuit (24 and 23, respectively). However, more than 50% of the participants did not like the taste of the pure (natural) and the raw Sochi seaweeds because of their fishy taste (Figure 2). It was also found that the appearance and the way the seaweeds were served affect the willingness of the individual to taste it. Moreover, the presence of a friend who likes to consume Seaweeds/*Spirulina* often encouraged the others to taste the algae.

4.2. Survey and research instrument

The following section presents the results of the second research method – the survey. The demographic results

Table 1. Socio-demographic profiling of the participants.

		Age	
Occupation			
Students	14	≤20	14
Administrative staff	10	20–40	6
Academic staff	5	40–60	10
Others	1	≥60	0
Gender		Nationality	
Male	18	Bahraini	20
Female	12	Non-Bahraini	10

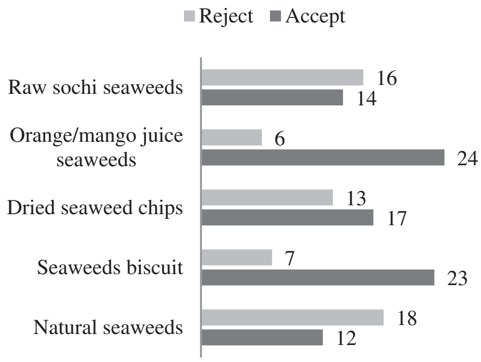


Figure 2. Experimental result of introducing to the participants algae in different ways.

Table 2. Socio-demographic profiling of the participants.

		Age	
Education			
Secondary school	78.2%	10–15	3%
BSc	13.9%	16–20	56%
MSc	6.7%	21–25	25%
PhD	0.6%	26–30	4%
Gender			
Male	81.4%	31–35	1%
Female	18.6%	>35	12%

of the responders show that 56% of the participants were 16–20 years old; most were from secondary school levels (78.2%), as shown in Table 2. The majority of the participants were male (80%). The high acceptance rate of consuming algal products among the young suggests that improving the diet in GCC region using algae would be possible.

The instrument used here was developed based on several studies addressing the perception of new technologies using TAM models. As such, scales for measuring sensory appeal and Perceived healthiness were developed by adapting items from the measurements of Radder and Roux (2005); Ng'ong'ola-Manani et al. (2014), Koller et al. (2015), Levine (2016) and Olasehinde et al. (2017). The scales of perceived risk and subjective norm were developed by adapting items from Choo et al. (2004), Bruhn (2007), Cardello et al. (2007) and Pande and Soodan (2015). Food neo-phobia and knowledge and experience were developed by adapting items from Loewen and Pliner (2000), Labrecque et al. (2006), El Dine and Olabi (2009) and Barcellos et al. (2010). The instruments were divided into two main sections, demographic and research variables. The demographic information of the participants includes Gender, age, weight, the level of education and health status. The second section of the questionnaire includes research variables such as sensory appeals, perceive healthiness of food, knowledge and experience/familiarity, subjective norm, perceived risk and uncertainty, food neo-phobia and behavioural intention to consume the algal product.

Table 3. Socio-demographics of the participants.

How do you evaluate your weight		
I am overweight		27.9%
I am underweight		18.2%
My weight is ideal		54%
Diseases you are suffering from		
Diabetes	Allergies	High blood pressure
81.1%	3%	10.9%
High cholesterol	High triglycerides	Osteoporosis
1.2%	1.8%	1.2%

Table 4. Behaviour regarding eating algal products.

Yes	No	
73%	27%	Do you like seafood (Fish, shrimps, … etc)?
90%	10%	Do you know what is meant by algae (Seaweeds)?
46%	55%	Have you ever heard about algae (Seaweeds) as alternative food?
71%	30%	Have you seen algae (Seaweeds)/algal food in the supermarket?
84.5%	14.5%	Have you ever seen one of your family member or friend eating algae (Seaweeds)?

Five Likert-type scales were used in the questionnaire, which ranges from strongly disagree (1) to strongly agree (5).

The results presented in Table 3 show that most of the participants felt that their weight is ideal (54%), while around 30% believe that they are overweight. Moreover, most of the participants are diabetic (81%) or have high blood pressure (11%), while relatively few suffered from high cholesterol or osteoporosis (1.2%).

Tables 3 and 4 show the participants' awareness and attitude towards the algae and seaweeds food. We found that 90% of the participants were aware of seaweeds. Moreover, 71% of them knew that seaweeds products were sold in the market and 85% saw their parents/friend using at least one kind of seaweed as food. However, almost 46% of the participants did not know that lots of algae can be eaten and 54% of them accepted to consume algae for the purpose of losing weight. Most importantly, 73% of the participants prefer to consume sea food.

4.3. Hypotheses testing

The statistical objective of PLS is to show high path coefficient and significant *t*-statistics. Therefore, the causal relationships in the research model were tested by applying a bootstrapping produce, standard error and *t*-statistics. This permits the measurement of the statistical significance of the path coefficients.

We found that sensory appeal ($\beta = 0.348$, $t = 5.767$), perceived healthiness ($\beta = 0.312$, $t = 4.903$) and knowledge and experience ($\beta = 0.303$, $t = 4.408$) have an indirect effect on the behavioural intention of the individual to consume algae via received risk and uncertainty (Table 5). Three of them show either

Table 5. Hypothesis testing (path analysis).

Hypotheses	*B*	*T*	Status
Sensory appeal – perceive risk and uncertainty	0.348	5.767	accepted
Perceived healthiness – perceive risk and uncertainty	0.312	4.903	accepted
Knowledge–experience–familiarity – perceive risk and uncertainty	0.303	4.408	accepted
Perceived risk and uncertainty – behavioral intention	0.414	4.459	accepted
Subjective norm – behavioural intention	0.255	2.806	accepted
Neo phobia –behavioural intention	0.369	5.946	accepted

Table 6. Explanation of variances.

Factor	R^2
Perceive risk and uncertainty	0.344
Behavioural intention	0.464

weak or insignificant direct impact on the behavioural intention ($\beta = 0.196$, $t = 2.582$, $\beta = 0.026$, $t = 0.267$, $\beta = 0.098$, $t = 1.582$). Thus, sensory appeal, perceived healthiness and knowledge, and experience are major factors in enhancing the perception of the risk and uncertainty of the algae food, explaining 34% of the variances on the perception of risk and uncertainty. We found that perceived risk and uncertainty ($\beta = 0.414$, $t = 4.459$), subjective norms ($\beta = 0.255$, $t = 2.806$) and food neo-phobia $\beta = 0.369$, $t = 5.946$) have a high and direct effect on the intention of individuals to consume algae or algal product, explaining 46% of the variances on the behavioural intentions, as shown in Table 6.

4.4. Assessment of model measurement

To test the research model, a PLS path analysis was performed using SmartPLS-3. The goodness-of-fit indexes (GFI) of latent variables are shown in Tables 7 and 8, indicating that the model has good fitness. Most of the AVE values are greater than 0.5, and all values of composite reliability are greater than 0.7, while all of the Cronbach's Alpha values are greater than 0.7.

5. Discussion

To the best of our knowledge, this study is the first to explore the consumer acceptance towards using algae as a food source in the Middle East. To assess the acceptance of eating algae, 30 individuals from UOB were given different types of algal food products(different types of pure dry Seaweeds, crispy Seaweeds, spicy Seaweeds, biscuits supplemented with Seaweeds and *Spirulina*), orange and mango juices mixed separately with Seaweeds. The majority of participants accepted eating biscuits baked with Seaweeds, Seaweeds chips and drinking orange/mango juice supplemented with seaweeds. A high

Table 7. AVE, composite reliability, and Cronbach's Alpha.

Construct	AVE	Composite Reliability	Cronbach's Alpha
Sensory appeal	0.670	0.875	0.973
Perceive health	0.798	0.754	0.799
Knowledge and experience/familiarity	0.824	0.953	0.957
Perceived risk and uncertainty	0.756	0.920	0.924
Food neophobia	0.650	0.876	0.876
Subjective norm	0.745	0.965	0.896
Behavioural intention	0.895	0.954	0.948

Table 8. Factor loading of the model constructs.

Construct	Items	Factor loading	Construct	Items	Factor loading
Sensory appeal	SA1	0.60	Perceived risk and uncertainty	PRU1	0.798
	SA2	0.754		PRU2	0.690
	SA3	0.856		PRU3	0.970
	SA4	0.835		PRU4	0.696
	SA5	0.976			
Perceive health	PH1	0.867	Food neophobia	FN1	0.967
	PH2	0.786		FN2	0.859
	PH3	0.657			
	PH4	0.785			
	PH5	0.687			
	PH6	0.987			
	PH7	0.897			
	PH8	0.946			
Knowledge and experience/ familiarity	KEF1	0.647	Subjective norm	SN1	0.798
	KEF2	0.675		SN2	0.897
	KEF3	0.970		SN3	0.687
	KEF4	0.869			
	KEF5	0.698			
	KEF6	0.796			
Behavioural Intention	BI1	0.689			
	BI2	0.708			

acceptance of using algae as food was shown for the juices, probably due to the diluted fishy taste of algae. On the other hand, there was significant reluctance to taste the dry pure Seaweeds (including the sushi seaweeds). Encouraged by our initial findings, we repeated the experiment on a larger scale atBIGS-2012, where a large number of people with a different background visit the UOB pavilion.

The visitors to the UOB pavilion in BIGS-2012 were of various positions and ages. In this pavilion, the author tried to prepare an Arabian food mixed with algae (i.e. different types of cheeses and bread baked with Seaweeds) plus other natural dried Seaweeds and *Spirulina*. The visitors reacted pleasantly and accepted consuming Arabian food supplemented with algae in addition to the naturally dried algae (seaweeds) and *Spirulina*, especially when they knew about their medical benefits. The visitors were happy to taste the algae and encouraged their friends and relatives to visit the UOB pavilion. Some of the visitors visited the pavilion several times on different days to re-taste the algal food products.

For the data collection, hard and online copies of questionnaires were used. In addition to the online questionnaire, paper questionnaires were distributed to the visitors to the UOB pavilion in BIGS-2012 to measure their willingness to consume algae.

An online questionnaire was selected for its cost advantage, greater geographical coverage and reduction of bias caused by the interviewer. Therefore, this study included the visitors to BIGS-2012, the individuals that participated in Experiment 2, and others from the Kingdom of Bahrain that participated online. The questionnaire was designed in two sections: deographics and research variables. In the demographics section, there was a total of five variables: gender, age, weight, the level of education and health status (Tables 1 and 2). The second section of the questionnaire includes research variables such as sensory appeals, perceive healthiness of food, knowledge and experience/familiarity, subjective norm, perceived risk and uncertainty, food neo-phobia and behavioural intention to consume the algal product. Five Likert-type scales were used in the questionnaire, which ranged from strongly disagree (1) to strongly agree (5). The sample size of this study was 300 individuals. According to Rezaei (2015), the minimum requirement of a sample size to test the model using PLS-SEM is 100–150 participants. Delice (2010) stated that larger sample size is needed to confer higher accuracy. Thus, the sample size of this study is in the acceptable range.

Socio-demographic variables normally shape food consumption (Moreira et al., 2010). The demographic results of the responders show that 56% of the participants were 16–20 years old. This age group easily accepts any change, including consuming unfamiliar food. The high acceptance rate of consuming algal products in the young ensures that improving the diet in the GCC region could be possible by introducing algae into their diet. Regarding the role of algae in losing weight, despite the ideal weight shown by more than half of the participants (54%), they accept that consuming algae has benefits, including aiding weight loss (Langea, Hausera, Nakamurab, & Kanayab, 2015).

In this study, sensory appeal (appearance, colour, texture, taste and smell) has been found to have a positive impact on perceived risk and uncertainty (hypothesis 1) which, in turn, affects behavioural intention, thus leading to the decision to consume algal products. This hypothesis was supported by several studies that confirm the role of sensory aspects of perception in food acceptance and, ultimately, the success of a product (Grunert, 2005; Grunert et al., 2003). In contrary, another study found that the consumer perceptions of food quality do not mainly depend on the sensory characteristics of the product but depend significantly on other factors such as cultural, social, cognitive and attitudinal factors related to the product and consumer (Cardello, 2003).

The effect of perceived healthiness of food on perceived risk and uncertainty was found to be significant in this research (hypothesis 2). This finding was consistent with other studies (Cardello, 2003; Poínhos et al., 2014; Ronteltap et al., 2007; Verdurme & Viaene, 2001) that have indicated a positive link between perceived healthiness, perceived risk, and uncertainty and behaviour. In addition, food safety concerns have increased considerably over the past decade with consumers becoming more aware of the possible health hazards associated with novel technologies in the food industry, such as the presence of preservative (nitrite) in meat, the use of nanoparticles in food, genetically engineered food and the use of radiation to sterilize food (Ajzen, 2005; Bord and O'Connor, 1990; Makatouni, 2001). Another study has confirmed that the health concerns of the consumer related to food hazards are significant determinants of acceptance (Miles & Frewer, 2001). When consumers knew about the negative health effect, which is associated with the use of nitrite in meat processing, they would likely select healthier meat products (Ajzen, 2005; Hung, de Kok, & Verbeke, 2016; Wilcock, Pun, Khanona, & Aung, 2004). Health consideration, natural content and weight control are important factors affecting the acceptance of seaweed consumption as a nutritive food (Prescott et al., 2002). Moreover, the perception of risk and uncertainty of novel food technologies will affect the behaviour of the consumer in selecting the food product (Ronteltap et al., 2007). Consumers who are able to control their own health status through their behaviours might be more encouraged to adopt consumer products with significant health benefits (Poínhos et al., 2014). Consequently, consumer perceptions of risk and uncertainty associated with food innovations play a critical role inedible algal product acceptance.

Here we show that knowledge and experience/familiarity has a positive impact on perceived risk and uncertainty (hypothesis 3). This means that consumers will be more aware of the risks of consuming algae when they are familiar with algae, know their benefits and have detailed, clear information, thus leading to an increase in the perception. The consumer's perception of food depends on available knowledge. The type of information needed could be packaging, nutritional value, the product's name, brands, labels, context and the situation in which the food is to be consumed. Food products have a remarkable effect on product preference, perceived sensory quality, acceptance, intended purchase and consumption (Cardello, 2003). Risk and uncertainty are critical factors affecting acceptance of food innovations, which leads to perceptions of risk and uncertainty (Cardello, 2003). The role of knowledge on perceived risk and uncertainty is supported by the findings of another study which proved that both experts and consumers expressed concerns about the potential risks associated with using nanotechnology to produce food and food products. Regarding the production of food via nanotechnology, experts perceived a greater risk than normal individuals as they are totally aware of the risks caused by food manipulated by nanotechnology. This demonstrates that the knowledge about any new technology will greatly affect the willingness to accept it.

The role of familiarity of consuming algae shown in this study is significant. A high percentage of the participants (85%) accepted consuming algae due to the presence of a friend or one member of their families consuming algae. Thus, the presence of a family member or a friend consuming algae enhances the acceptance of using algae as food. The high acceptance of consuming algae was likely due to the popularity of eating fish in this region (we found that 73% of the participants consume fish). Familiarity with a food is highly positively correlated with the behavioural intention. For example, Asian people that migrated to America and Europe introduced seaweeds to those regions. The presence of Asians who consume algae and the availability of seaweeds in America and Europe inspire confidence in citizens of those regions, resulting in an increase in acceptance of consuming seaweeds. Thus, other populations have adopted algae in their food due to familiarity. Moreover, more than half of the participants were young (54%), which means that they are accepting of change, especially when it is associated with diet. Although 46%of the participants showed primary ignorance about what is meant by algae, this ignorance did not hinder their acceptance to use algae as a food source. Therefore, there are factors other than the knowledge that will affect the acceptance of food.

We found that sensory appeal ($\beta = 0.348$, $t = 5.767$), perceived healthiness ($\beta = 0.312$, $t = 4.903$) and knowledge and experience ($\beta = 0.303$, $t = 4.408$) have an indirect effect on the behavioural intention of the individual to consume algae via received risk and uncertainty (Table 5). Therefore, sensory appeal, perceived healthiness, knowledge and experience are the main factors enhancing the individual perception of the risk and uncertainty of consuming algal food products. On the other hand, those factors show either weak or insignificant direct impact on the behavioural intention ($\beta = 0.196$, $t = 2.582$, $\beta = 0.026$, $t = 0.267$, $\beta = 0.098$, $t = 1.582$). Thus, sensory appeal, perceived healthiness and knowledge and experience/familiarities are the principal factors for enhancing the individual perception of the risk and uncertainty of algal food, although they explain just 34% of the observed variances. This means that there are other factors affecting the perceived risk and uncertainty that were not addressed here (e.g. nutritional value, quality, locality and price) and that should be investigated in future studies.

The results revealed that perceived risk and uncertainty ($\beta = 0.414$, $t = 4.459$), subjective norms ($\beta = 0.255$, $t = 2.806$) and food neo-phobia ($\beta = 0.369$, $t = 5.946$) have a high and direct effect on the intention of individuals to consume algae or algal products. However, these explain just 46% of the variances on the behavioural intentions (Table 6). Perceived risk and uncertainty were shown to have a positive impact on behavioural intention (hypothesis 4), which is measured by choice, purchase, and consumption (Cardello, Schuts, & Lesher, 2000). Expectations of liking/disliking a food can be affected by a variety of appropriate factors that are independent of the food itself. Those factors are societal, contextual, cultural, psychological and economical (Lambros et al., 2014; Higgs & Thomas, 2016). These factors should be studied further.

Subjective norm has been found to have a significant positive relationship with intention to consume algae product (Hypothesis 5). This effect of the subjective norm was supported by the study of Hasbullah et al. (2016) who detected a positive relationship between subjective norm and intention to buy online. Moreover, it is supported by Theory Reasoned Action (TRA), which confirms a significantly positive relationship between subjective norm and behaviour (Fishbein & Ajzen, 1975). In other words, the friends or family of individuals apply pressure, which increases the person's purchase intention. Subjective norm was considered as a strong determinant of intention (Karaiskos, Tzavellas, Balta, & Paparrigopoulos, 2010).

Consumer fears and risk perceptions have received considerable attention in the studies related to consumer acceptance of novel food (Miles & Frewer, 2001; Cardello, 2003). The current study found that food neo-phobia has a positive impact on behavioural intention (Hypothesis 6). Some individuals will reject eating unfamiliar food due to food neo-phobia. Earlier studies have confirmed distinct food neo-phobia effects (Pliner & Hobden, 1992; Tuorila, Meiselman, Bell, Cardello, & Johnson, 1994). This fear response may be due to sensory appeal, presentation, risk or food unfamiliarity. Expert opinions might reduce food neo-phobia as it increases potential acceptability by consumers (Giles, Kuznesof, Clark, Hubbard, & Frewer, 2015).

By adopting a TAM model, this study has highlighted a major gap in our knowledge of consumer perceptions, attitudes, beliefs and expectations in the community of Bahrain, which is similar to other GCC regions. As expected, seaweed will gain greater acceptance in regions such as the Middle East, where they have more common food habits in addition to the spread of Asian foods due to the large Asian communities in the region. Our data will provide policy makers, marine specialists and the food industry with evidence of the consumer acceptance of algal food products. Moreover, the result will provide evidence that will assist the main stakeholders making fine-tuning of policies, in addition to the estimation of consumers reaction in future towards algal food products.

6. Conclusions

This study revealed that the community of Bahrain is aware of the medical and nutritional importance of using algae as a source of food. Moreover, sensory appeal, perceived healthiness and knowledge/experience have a direct effect on the behavioural intention. There are other factors that should now be studied. In addition, perceived risk and uncertainty, subjective norm and food neo-phobia have a high and direct effect on the intention of the individual to consume algae. The results of this study will encourage the people in the Arabian Gulf to consider algae as an alternative food source. This influence will extend to the Middle East. In addition, our data will encourage the researchers in the field of nutrition to study the edible algae in the Arabian Gulf seas in terms of their nutritional importance. The people in the Kingdom of Bahrain are ready to enhance their diet with algae. Knowing that the people are accepting to use algae as a source of food will enhance the algal food business in the area.

7. Implications

This study provides suggestions for policy makers and marine specialists who are associated with food sector. In addition, the study can be helpful for the food industry to identify their target consumers by showing the effect of demographic factors, food neophobia, health, perceived risk and uncertainty, and knowledge/familiarity with algal food product market. Consumer's acceptance of emerging technologies and their applications is a critical determinant of successful commercialization, and the strategy of the marketers should be altered accordingly. This study shows that health is a critical factor of acceptance. Therefore, retailers should concentrate on this factor when marketing to attract potential consumers. Moreover, the cultivation of algae for the purpose of producing food, medicines and cosmetics in the GCC should be considered in the market strategy.

Acknowledgements

It is my pleasure to acknowledge Dr. Jaflah Al-Ammari for her generous help without which the completion of the current study would not have been possible.

Disclosure statement

No potential conflict of interest was reported by the author.

References

AbuQamar, S., Alshannag, Q., Sartawi, A., & Iratni, R. (2015). Educational awareness of biotechnology issues among undergraduate students at the United Arab Emirates University. *Biochemistry and Molecular Biology Education*, *43*(4), 283–293. doi: 10.1002/bmb.20863.

Ajzen, I. (2005). *Attitudes, personality, and behavior*. Berkshire, UK: Open University Press.

Al-Amoudi, O. A. (2009). Chemical composition and antioxidant activities of Jeddah corniche algae, Saudi Arabia. *Saudi Journal of Biological Sciences*, *16*, 23–29. doi: 10.1016/j.sjbs.2009.07.004.

Al-Harthi, M. A., & Al-Deek, A. (2011). The effects of preparing methods and enzyme supplementation on the of brown marine algae (*Sargassum dentifebium*) meal in the diet of laying hens. *Italian Journal of Animal Science*, *10*, e48. doi: 10.4081/ijas.2011.e48.

Barcellos, M. D., Kügler, J. O., Grunert, K. G., Van Wezemael, L., Pérez-Cueto, F. J. A., Ueland, Ø., & Verbeke, W. (2010). European consumers' acceptance of beef processing technologies: A focus group Study. *Innovative Food Science and Emerging Technologies*, *11*, 721–732. doi: 10.1016/j.ifset.2010.05.003.

Bord, R. J. & O'Connor, R. E. (1990). Risk communication, knowledge, and attitudes: Explaining reactions to a technology perceived as risky. *Risk Analysis*, *10*, 499–506. doi: 10.1111/j.1539-6924.1990.tb00535.x.

Bouyer, M., Bagdassarian, S., Chaabanne, S., & Mullet, E. (2001). Personality correlates of risk perception. *Risk Analysis*, *21* (3), 457–465. doi: 10.1111/0272-4332.213125.

Bruhn, C. (2007). Enhancing consumer acceptance of new processing technologies. *Innovative Food Science and Emerging Technologies*, *8*, 555–558. doi: 10.1016/j.ifset.2007.04.006.

Cardello, A. V. (2003). Consumer concerns and expectations about novel food processing technologies: Effects on product linking. *Appetite*, *40*, 217–233. doi: 10.1016/S0195-6663(03)00008-4.

Cardello, A. V., Schuts, H. S., & Lesher, L. (2000). Predictors of food acceptance, consumption and satisfaction in specific eating situations. *Food Quality and Preference*, *11*, 201–216. doi: 10.1016/S0950-3293(99)00055-5.

Cardello, A. V., Schutz, H. J., & Lesher, L. L. (2007). Consumer perceptions of foods processed by innovative and emerging technologies: A conjoint analytic study. *Innovative Food Science and Emerging Technologies*, *8*, 73–83. doi: 10.1016/j.ifset.2006.07.002.

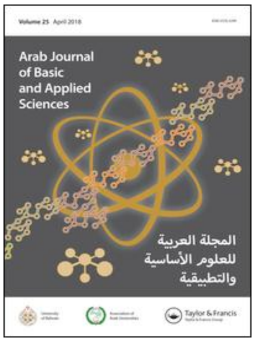
Choo, H., Chung, J., & Pysarchik, D. T. (2004). Antecedents to new food product purchasing behavior among innovator groups in India. *European Journal of Marketing*, *38* (5/6), 608–625. doi: 10.1108/03090560410529240.

Cordain, L., Eaton, S. B., Miller, J. B., Mann, N. & Hill, K. (2002). The paradoxical nature of hunter-gatherer diets: Meat-based, yet non-atherogenic. *European Journal of Clinical Nutrition*, *56*, S42–S52. doi: 10.1038/sj.ejcn.1601353.

Costa, A. I. A. & Jongen, W. M. F. (2006). New insights into consumer-led food product development. *Trends in Food*

Science & Technology, *17*(8), 457–465. doi: 10.1016/j.tifs.2006.02.003.
Delice, A. (2010). The sampling issues in quantitative research [Kuram ve Uygulamada Eğitim Bilimleri]. *Educational Sciences: Theory& Practice*, *10* (4), 1969–2018.
El Dine, A. N. & Olabi, A. (2009). Effect of reference foods in repeated acceptability tests: Eating familiar and novel foods using 2 acceptability scales. *Journal of Food Science*, *74*(2), S97–S106. doi: 10.1111/j.1750-3841.2008.01034.x.
El-Deek, A. A., Al-Harthi, M. A., Abdalla, A. A. & Elbanoby, M. M. (2011). The use of brown algae meal in finisher broiler diets. *Egyptian Poultary Science*, 31 (IV), 767–781.
FAO. (2003). *A guide to the seaweed industry*. Rome: FAO Fisheries and Aquaculture Department.
Fishbein, M. & Ajzen, I. (1975). *Belief, attitude, intention, and behavior: An introduction to theory and research*. Reading, MA: Addison-Wesley.
Freije, A. M., Hussain, T., & Salman, E. A. (2017). Global warming awareness among the University of Bahrain science students. *Journal of the Association of Arab Universities of Basic and Applied Sciences*, *22*, 9–16. doi: 10.1016/j.jaubas.2016.02.002.
Giles, E. L., Kuznesof, S., Clark, B., Hubbard, C., & Frewer, L. J. (2015). Consumer acceptance of and willingness to pay for food nanotechnology: A systematic review. *Journal of Nanoparticle Research*, *17* (467), 1–26. doi: 10.1007/s11051-015-3270-4.
Grunert, K. G. (2005). Consumer behaviour with regard to food innovations: Quality perception and decision-making. In W. M. F. Jongen and M. T. G. Meulenberg (Eds.), *Innovation in agri-food system*. Washington, The Netherlands: Washington University press.
Grunert, K. G., Bredahl, L., & Scholderer, J. (2003). Four questions on European consumers' attitudes toward the use of genetic modification in food production. *Innovative Food Science and Emerging Technologies*, *4*, 435–445. doi: 10.1016/S1466-8564(03)00035-3.
Grunert, K. G. & Valli, C. (2001). Designer-made meat and dairy products: Consumer-led product development. *Livestock Production Science*, *72*, 83–98. doi: 10.1016/S0301-6226(01)00269-X.
Gutiérrez-Salmeán, G., Fabila-Castillo, L., & Chamorro-Cevallos, G. (2015). Nutritional and toxicological aspects of Spirulina (*Arthrospira*). *Nutricion Hospitalaria*, *32*, 34–40. doi: 10.3305/nh.2015.32.1.9001.
Hasbullah N. A., Osmanb, A. H, Abdullah, S., Salahuddind, S. N., Ramlee N. F., & Soha, M. H. (2016). The relationship of attitude, subjective norm and website usability on consumer intention to purchase online: An evidence of Malaysian youth. *Procedia Economics and Finance*, *35*, 493–502. doi: 10.1016/S2212-5671(16)00061-7.
Henrikson, R. (2009). *Earth food Spirulina: The complete guide to a powerful new food that can help rebuild our health and restore our environment*. San Rafael: Ronore Enterprises, Inc.
Higgs, S. & Thomas, J. (2016). Social influences on eating. *Current Opinion in Behavioural Sciences*, *9*, 1–6. doi: 10.1016/j.cobeha.2015.10.005.
Hung, Y., de Kok, T. M., & Verbeke, W. (2016). Consumer attitude and purchase intention towards processed meat products with natural compounds and a reduced level of nitrite. *Meat Science*, *121*, 119–126. doi: 10.1016/j.meatsci.2016.06.002.
IIMSAM (Intergovernmental Institution for the Use of Micro-Algae Spirulina Against Malnutrition). 2008. *IIMSAM works. With concrete deeds. A year in review support IIMSAM support life*. New York: IIMSAM.
Ismail, A. & Hong, T. S. (2002). Antioxidant activity of selected commercial seaweeds. *Malaysian Journal of Nutrition*, *8*(2), 167–177.
Jacqueline, J. K. & Julie, A. L. (2002). The influence of culture on consumer impulsive buying behavior. *Journal of Consumer Psychology*, *2* (2), 163–176. doi: 10.1207/S15327663JCP1202_08.
Karaiskos, D., Tzavellas, E., Balta, G., & Paparrigopoulos, T. (2010). Social network addiction: A new clinical disorder? *European Psychiatry*, *25*, 855. doi: 10.1016/S0924-9338(10)70846-4.
Kent, M., Welladsen, H. M., Mangott, A., & Li, Y. (2015). Nutritional evaluation of Australian microalgae as potential human health supplements. *PLoS One*, *10*, 1–14. doi: 10.1371/journal.pone.0118985.
Khotimchenko, S. V. & Kulikova, I. V. (2000). Lipids of different parts of the lamina of *Laminaria japonica* Aresch. *Botanica Marina*, *43*, 87–91. doi: 10.1515/BOT.2000.008
Kılınç, B., Cirik, S., Turan, G., Tekogul, H., & Koru, E. (2013). *Seaweeds for food and industrial applications: Food industry*. Croatia: InTech.
Koller, M., Salernob, A., & Braunegg, G. (2015). Value-added Products from Algal Biomass. In A. Perosa, G. Bordignon, G. Ravagnan, & S. Zinoviev (Eds.), *Algae as a potential source of food and energy in developing countries sustainability, technology and selected case studies* (Vol. 2, pp. 19–40). Edizioni Ca'Foscari: Scienza e società.
Labrecque, J., Doyon, M., Bellavance, F., & Kolodinsky, J. (2006). Acceptance of functional foods: A comparison of French, American, and French Canadian consumers. *Canadian Journal of Agricultural Economics*, *54* (11), 647–661. doi: 10.1111/j.1744-7976.2006.00071.x.
Lahteenmaki, L., Grunert, K. G., Astrom, A., Ueland, O., Arvola, A., & Bech-Larsen, T. (2002). Acceptability of genetically modified cheese presented as real product alternative. *Food Quality and Preference*, *13*, 523–534. doi: 10.1016/S0950-3293(01)00077-5.
Lambros, T., Karasavvoglou, A., Tsourgiannis, C. A., Florou, G., Theodosiou, T., & Valsamidis, S. (2014). Factors affecting consumers in Greece to buy during the economic crisis period food produced domestically in Greece. The Economies of Balkan and Eastern Europe Countries in the Changed World (EBEEC 2013). *Procedia Economics and Finance*, *9*, 439–455. doi: 10.1016/S2212-5671(14)00046-X.
Langea, K. W., Hausera, J., Nakamurab, Y., & Kanayab, S. (2015). Dietary seaweeds and obesity. *Food Science and Human Wellness*, *4* (3), 87–96. doi: 10.1016/j.fshw.2015.08.001.
Levine, L. (2016). Algae: A way of life and health. In J. Fleurence & I. Levine (Eds.), *Seaweed in health and disease prevention* (pp. 1–5). New York: Academic Press, Elsevier.
Loewen, R. & Pliner, P. (2000). The Food Situations Questionnaire: A measure of children's willingness to try novel foods in stimulating and non-stimulating situations. *Appetite*. *35*, 239–250. doi: 10.1006/appe.2000.0353.
Maehre, H. K., Malde, M. K., Eilertsen, K. E., & Elvevoll, E. O. (2014). Characterization of protein, lipid and mineral contents in common Norwegian seaweeds and evaluation of their potential as food and feed. *Journal of the Science of Food and Agriculture*, *94*(15), 3281–3290. doi: 10.1002/jsfa.6681.

Makatouni, A. (2001). What motivates consumers to buy organic food in the UK? *British Food Journal*, *104* (3–5), 345–352. doi: 10.1108/00070700210425769.
Makki, M. (2014). Lebanon algae species: Promising source of protein, fuel. *Nature*. doi:10.1038/nmiddleeast.2014.245.
Matondo, F. K., Takaisi, K., Nkuadiolandu, A. B., Lukusa, A. K., & Aloni, M. N. (2016). *Spirulina* supplements improved the nutritional status of undernourished children quickly and significantly: Experience from Kisantu, the Democratic Republic of the Congo. *International Journal of Pediatrics*, *2016*, 1296414. doi: 10.1155/2016/1296414.
Miles, S. & Frewer, L. J. (2001). Investigating specific concerns about different food hazards. *Food Quality and Preference*, *12*, 47–61. doi: 10.1016/S0950-3293(00)00029-X.
Moreira, P., Santos, S., Padrão, P., Cordeiro, T., Bessa, M., Valente, H., …., Moreira, A. (2010). Food patterns according to sociodemographics, physical activity, sleeping and obesity in Portuguese children. *International Journal of Environmental Research and Public Health*, *7*, 1121–1138. doi: 10.3390/ijerph7031121.
Musaiger, A., Hassan A. S. & Obeid, O. (2011). The Paradox of nutrition-related diseases in the Arab countries: The need for action. *International Journal of Environmental Research and Public Health*, *8*(9), 3637–3671. doi: 10.3390/ijerph8093637.
Ng'ong'ola-Manani, T. A., Mwangwela, A. M., Schüller, R. B., Østlie, H. M., & Wicklund, T. (2014). Sensory evaluation and consumer acceptance of naturally and lactic acid bacteria-fermented pastes of soybeans and soybean–maize blends. *Food Science and Nutrition*, *2* (2), 114–131. doi: 10.1002/fsn3.82.
Olasehinde, T. A., Olaniran, A. O., & Okoh, A. I., 2017. Therapeutic potentials of microalgae in the treatment of Alzheimer's disease. *Molecules*, *22* (480), 1–8. doi: 10.3390/molecules22030480.
Pande, A. C. & Soodan, V. (2015). Role of consumer attitudes, beliefs and subjective norms as predictors of purchase behaviour: A study on personal care purchases. *The Business and Management Review*, *5* (4), 284–291.
Pliner, P. & Hobden, K. (1992). Development of a scale to measure the trait of food neophobia in humans. *Apetite*, *19*(2), 105–120. doi: 10.1016/0195-6663(92)90014-W.
Poínhos, R., van der Lans, I. A., Rankin, A., Fischer, A. R. H., Bunting, B., Kuznesof, S., …, Frewer, L. J. (2014). Psychological determinants of consumer acceptance of personalised nutrition in 9 European countries. *PLoS One*, *9* (10), e110614. doi: 10.1371/journal.pone.0110614.
Pollard, J., Kirk, S. F., & Cade, J. E. (2002). Factors affecting food choice in relation to fruit and vegetable intake: A review. *Nutrition Research Reviews*, *15* (2), 373–387. doi: 10.1079/NRR200244.
Prescott, J., Young, O., O'Neill, L., Yauc, N. J. N., & Stevensd, R., 2002. Motives for food choice: A comparison of consumers from Japan, Taiwan, Malaysia and New Zealand Steven. *Food Quality and Preference*, *13*, 489–495. doi: 10.1016/S0950-3293(02)00010-1.
Radder, L. & Roux, R. L. (2005). Factors affecting food choice in relation to venison: A south African example. *Meat Science*, *71*, 538–589. doi: 10.1016/j.meatsci.2005.05.003.
Rezaei, S. (2015). Segmenting consumer decision-making styles (CDMS) toward marketing practice: A partial least squares (PLS) path modeling approach. *Journal of Retailing and Consumer Services*, *22*, 1–15. doi: 10.1016/j.jretconser.2014.09.001.
Rimal, A. P., McWatters, K. H., Hashim, I. B., & Fletcher, S. M., 2004. Intended vs. actual purchase behavior for irradiated beef: A simulated supermarket setup experiment. *Journal of Food Products Marketing*, *10* (4), 1–15. doi: 10.1300/J038v10n04_01.
Ronteltap, A., Trijp, J. C. M, Renses, R. J., & Frewer, L. J. (2007). Consumer acceptance of technology-based food innovations: Lessons for the future of nutrigenomics. *Appetite*, *49*, 1–17. doi: 10.1016/j.appet.2007.02.002
Sańchez-Machado, D. I., Lo ́pez-Cervantes, J., Lo ́pez-Hernańdez, J., & Paseiro-Losada, P. (2004). Fatty acids, total lipid, protein and ash contents of processed edible seaweeds. *Food Chemistry*, *85*, 439–444. doi: 10.1016/j.foodchem.2003.08.001.
Sabbe, S., Verbeke, W., Deliza, R., Matta, V., & Van Damme, P. (2009). Effect of a health claim and personal characteristics on consumer acceptance of fruit juices with different concentrations of açaí (*Euterpe oleracea* Mart.) *Appetite*, *53*(1):84–92. doi: 10.1016/j.appet.2009.05.014.
Siegrist, M. (1998). Belief in gene technology: The influence of environmental attitudes and gender. *Personality and Individual Differences*, *24* (6), 861–866. doi: 10.1016/S0191-8869(98)00021-X.
Stévant, P., Rebours, C., & Chapman, A. (2017). Seaweed aquaculture in Norway: Recent industrial developments and future perspectives. *Aquaculture International*, *25*(4), 1373–1390. doi: 10.1007/s10499-017-0120-7.
Tenbült, P., de Vries, N. K., Dreezens, E., & Martijn, C. (2008). Intuitive and explicit reactions towards "new" food technologies: Attitude strength and familiarity. *British Food Journal*, *110* (6), 622–635. doi: 10.1108/00070700810877924.
Thomas, D. N. (2002). *Seaweeds*. Washington, DC: Smithsonian Institution Press.
Tuorila, H., Meiselman, H. L., Bell, R., Cardello, A. V., & Johnson, W. 1994. Role of sensory and cognitive information in the enhancement of certainty and liking for novel and familiar foods. *Appetite*, *23*, 231–246. doi: 10.1006/appe.1994.1056 .
USDA, Department of Agriculture. (1992). *Human nutrition information service*. Washington, DC: USDA.
Verbeke, W. (2005). Consumer acceptance of functional foods: Socio-demographic, cognitive and attitudinal determinants. *Food Quality Preference*, *16*, 45–57. doi: 10.1016/j.foodqual.2004.01.001.
Verdurme, A. & Viaene, J. (2001). Consumer attitudes towards gm food: Literature review and recommendations for effective communication. *Journal of International Food & Agribusiness and Marketing*, *13* (2/3), 77–98. doi: 10.1300/J047v13n02_05.
Vidigal, M. C. T. R., Minim, V. P. R., Simiqueli, A. A., Souza, P. H. P., Balbino, D. F., & Minim, L. A. (2015). Food technology neophobia and consumer attitudes toward foods produced by new and conventional technologies: A case study in Brazil. *LWT-Food Science and Technology*, *60* (2), 832–840. doi: 10.1016/j.lwt.2014.10.058.
Wilcock, A., Pun, M., Khanona, J., & Aung, M. (2004). Consumer attitudes, knowledge and behaviour: A review of food safety issues. *Trends in Food Science & Technology*, *15*, 56–66. doi: 10.1016/j.tifs.2003.08.004.



Taylor & Francis
Taylor & Francis Group

Arab Journal of Basic and Applied Sciences

ISSN: (Print) 2576-5299 (Online) Journal homepage: http://www.tandfonline.com/loi/tabs20

Modulation of cytokine production from cultured mononuclear cells of leukemia patients by *Hypericum triquetrifolium* Turra methanolic extract

Ali H. Ad'hiah, Ruqaia M. Ibraheem & Khulood W. Abbood

To cite this article: Ali H. Ad'hiah, Ruqaia M. Ibraheem & Khulood W. Abbood (2018) Modulation of cytokine production from cultured mononuclear cells of leukemia patients by *Hypericum triquetrifolium* Turra methanolic extract, Arab Journal of Basic and Applied Sciences, 25:1, 13-19, DOI: 10.1080/25765299.2018.1449413

To link to this article: https://doi.org/10.1080/25765299.2018.1449413

Accepted author version posted online: 09 Apr 2018.
Published online: 18 Apr 2018.

ARAB JOURNAL OF BASIC AND APPLIED SCIENCES
2018, VOL. 25, NO. 1, 13–19
https://doi.org/10.1080/25765299.2018.1449413

University of Bahrain · Taylor & Francis · Taylor & Francis Group

ORIGINAL ARTICLE

OPEN ACCESS · Check for updates

Modulation of cytokine production from cultured mononuclear cells of leukemia patients by *Hypericum triquetrifolium* Turra methanolic extract

Ali H. Ad'hiah[a], Ruqaia M. Ibraheem[b] and Khulood W. Abbood[b]

[a]Tropical-Biological Research Unit, College of Science, University of Baghdad, Baghdad, Iraq; [b]College of Applied Biotechnology, Al-Nahrain University, Baghdad, Iraq

ABSTRACT

The effect of *Hypericum triquetrifolium* Turra (Family: Hypericaceae) methanolic extract on an *in vitro* cytokine release (IL-2, IL-4, IL-10, IL-17A and IFN-γ) from cultured mononuclear cells was determined. The cells were obtained from acute lymphoblastic (ALL) and acute myelogenous leukemic (AML) Iraqi patients. Two concentrations (1.146 and 1.719 µg/ml) of the extract were tested. They were correspondent to 20 and 30%, respectively of the plant IC_{50} (5.73 µg/ml). Chemical analysis of the extract showed its richness in flavonoids (115.73 µg/ml). Assessments of cytokine levels in supernatants of cultured cells revealed that ALL, AML or control cells responded differently to the plant extract in their production of IL-2, IL-10, IL-17A and IFN-γ, but leukemic cells were better than control cells in their response, while there was no effect on IL-4 production. The results suggested that *H. triquetrifolium* methanolic extract exerted immunomodulatory effects on cultured cells.

ARTICLE HISTORY
Received 1 December 2016
Accepted 2 October 2017

KEYWORDS
Hypericum triquetrifolium; acute lymphoblastic leukemia; acute myelogenous leukemia; cytokines

1. Introduction

The immune response modulation, as a possible therapeutic strategy by using medicinal plants or their secondary metabolites, has become a fruitful subject for scientific investigations. They are traditionally used in the treatment of various ailments and considered to be superior over the conventionally employed medicines, which are known to have unfavorable side effects (Chouhan, Islamuddin, Sahal, & Afrin, 2014). One of these plants is *Hypericum triquetrifolium* Turra (Family: Hypericaceae). It is a promising medicinal plant from Eastern Europe and the Mediterranean area, and traditionally used for its sedative, anti-helminthic, anti-inflammatory and antiseptic effects (Saad, Azaizeh, & Said, 2005). In addition, several studies have reported the potential use of its essential oil and crude extracts as therapeutic substances, mainly in the treatment of burns and gastroenteritis, and as anti-nociceptive and anti-oxidant drug. This species also has healing and diuretic properties and is used to treat kidney, urinary bladder, liver and migraine complications (Conforti, Loizzo, Statti, & Menichini, 2007). In addition, several biological potentials have described for different extracts of *H. triquetrifolium* or its natural products; for instance, anti-oxidant, anti-microbial, immunological, anti-mutagenic and anti-tumor potentials (Rouis et al., 2013; Saad et al., 2011).

Acute lymphoblastic leukemia (ALL) and acute myelogenous leukemia (AML) are a clonal malignancies, in which hematopoietic differentiation is profoundly blocked, and as a consequence, an overproduction of immature blasts is overwhelmed. During the genesis of these aberrant cells, some properties are established that enable leukemia cells to survive and proliferate, and counteract growth signals mediated by immunological mediators, such as cytokines (van Etten, 2007). Recent investigations also suggest that cytokines play an important role in progression of leukemia, and their modulation may have a therapeutic potential (Chiarini et al., 2015).

One of the approaches that are employed to determine immunological effects of medicinal plants or their products is assessment of cytokine production *in vitro* and *in vivo* (Saxena et al., 2016). Cytokines are low molecular weight glycoproteins produced by a number of cell types, predominantly leukocytes that regulate immunity, inflammation and hematopoiesis (Oppenheim, 2013). They are produced from various cells during the effector phases of immune responses and regulate a number of physiological and pathological functions including innate immunity, acquired immunity and inflammatory responses (Levine, 2013). However, none of the *Hypericum* species have been investigated in leukemic cells to determine their effects on cytokine

CONTACT Ali H. Ad'hiah dr.a.h.adhiah@gmail.com, dr.ahadhiah@sc.uobaghdad.edu.iq Tropical-Biological Research Unit, College of Science, University of Baghdad, Baghdad, Iraq

production by these cells, but PubMed searching revealed the existence of 16 studies that tested the potential of different species of the genus *Hypericum* on cytokine production from cells of different origins, and encouraging results were presented (Canning et al., 2010; Hu et al., 2006; Novelli et al., 2014; Saad et al., 2011). Accordingly, a study was designed to determine the effect of *H. triquetrifolium* methanolic extract on an *in vitro* cytokine release (IL-2, IFN-γ, IL-4, IL-10 and IL-17A) from cultured mononuclear cells obtained from ALL and AML Iraqi patients.

2. Materials and methods

2.1. Reagents

Aluminum chloride (BDH, UK), fetal calf serum, histopaque, penicillin, phytohemagglutinin, RPMI-1640 medium, rutin, streptomycin (Sigma, St. Louis, MO), heparin (Leo Pharmaceutical, Denmark), methanol (Fluka, Switzerland) and trypan blue (Pharmacia Fine chemical, Sweden) were purchased from their respective companies.

2.2. Plant collection and extraction

Dr. Saman Abdulrahman Ahmad (Department of Field Crops, College of Agriculture Science, University of Sulaymaniyah, Iraq) supplied the leaves of *H. triquetrifolium* as a powdered dried material. The plant was collected from the mountain regions in Tasloga (Sulaymaniyah); a city located 330 km north-east the capital Baghdad. The leaf powder was subjected to extraction with methanol (Fu et al., 2010). Briefly, 50 grams of the plant leaf powder were extracted with 80% methanol (250 ml) at 65 °C for 3 hours using the soxhlet apparatus. The extract solution was concentrated to dryness under a reduced pressure in a rotary evaporator to yield a dried crude extract, which was frozen at −20 °C until use to prepare the required concentrations. Two concentrations (1.146 and 1.719 μg/ml) of the extract were tested. They were correspondent to 20 and 30%, respectively of the IC_{50} (5.73 μg/ml) of *H. triquetrifolium* methanolic extract that was tested in four cancer cell lines and one normal primary cell culture (Conforti et al., 2007).

2.3. Determination of flavonoids

Total flavonoids content was spectrophotochemically determined in the methanolic extract of *H. triquetrifolium* as a rutin (flavonoid standard) equivalent by the aluminium chloride colorimetric method (Sakanaka, Tachibana, & Okada, 2005).

2.4. Leukemia patients and controls

The ethical committee at the Iraqi Ministry of Health approved the study. The leukemia patients (ALL and AML; each of 8 cases) were referred to the Baghdad Teaching Hospital for diagnosis and treatment. The diagnosis was based on a clinical examination and laboratory evaluations, which were carried out by the consultant medical staff at the hospital. The patients were Iraqi Arabs, and their age range was 25–40 years. They were firstly diagnosed, and none of them was under therapy. A further 7 apparently healthy controls were also investigated. They were university staff and students who had no history or signs of leukemia, and matched patients for ethnicity and age.

2.5. Isolation of mononuclear cells

Peripheral blood (8–10 ml) was obtained under aseptic conditions from each participant by a venipuncture, and distributed into two aliquots. The first (4 ml) was dispensed in a plain tube and used for the collection of serum, while the second aliquot was dispensed in a test tube containing heparin (10 IU/ml) and used for the isolation of mononuclear cells, which were enriched by a density-gradient centrifugation, using histopaque as a separating medium. After enrichment, the isolated cells were suspended in RPMI-1640 medium, cell viability was assessed by a dye exclusion test (trypan blue) and the count was adjusted to $2–3 \times 10^6$ cells/ml (Ad'hiah, 1990).

2.6. Experimental design

The experiments were designed to assess *in vitro* production of cytokines from cultured mononuclear cells obtained from ALL and AML patients and controls. To achieve such aim, six cultures were set-up for each participant. In culture I, cells were not treated with any material (negative control), while in culture II, cells were stimulated with phytohemagglutinin (PHA) only. For cultures III and IV, cells were treated with 1.146 and 1.791 μg/ml of plant extract, respectively and stimulated with PHA. In case of cultures V and VI, the cells were only treated with the two concentrations of plant extract, respectively.

2.7. Culture establishment

Following the experimental design presented in section 2.6, counted cells (1 ml) were transferred to a test tube, to which 0.1 ml of PHA and/or a plant extract was added. The tube was incubated at 37 °C for 48 hours (5% CO_2 and 80% relative humidity). After incubation, the tube was centrifuged (2000 rpm

Table 1. Effect of *H. triquetrifolium* methanolic extract on IL-2 production from cultured mononuclear cells of ALL and AML patients.

Culture number	Plant extract concentration (μg/ml)	IL-2 level Mean ± SD (pg/ml) ALL (No. = 8)	Controls (No. = 7)	AML (No. = 8)	*p* Value* ≤
I (Untreated)	–	7.4 ± 1.7^{E}	2.8 ± 1.5^{B}	5.7 ± 2.8^{E}	0.05
II (PHA only)	–	10.4 ± 3.8^{D}	5.9 ± 3.4^{B}	10.3 ± 3.3^{D}	0.05
III (PHA + Extract)	1.146	23.1 ± 1.6^{C}	12.8 ± 3.9^{A}	11.7 ± 1.7^{D}	0.01
IV (PHA + Extract)	1.791	36.4 ± 1.8^{A}	11.5 ± 1.5^{A}	25.3 ± 1.9^{A}	0.001
V (Extract only)	1.146	27.2 ± 1.9^{C}	12.4 ± 3.1^{A}	16.2 ± 1.8^{C}	0.001
VI (Extract only)	1.791	30.8 ± 1.1^{B}	14.4 ± 1.5^{A}	21.1 ± 2.6^{B}	0.001
Serum		23.7 ± 1.9^{C}	10.2 ± 3.2^{C}	28.2 ± 3.9^{A}	0.001

ALL: acute lymphoblastic leukemia; AML: acute myelogenous leukemia; PHA: phytohemagglutinin; SD: standard deviation; Different superscript letters represent a significant difference ($p \le 0.05$) between means of columns (cultures I–VI and serum).
*Probability of difference between means of ALL, controls and AML.

for 5 minutes) and the supernatant was collected, distributed into aliquots, and frozen at −20 °C until assessment of cytokines.

2.8. Assessment of cytokines

Sera and cell culture supernatants of leukemia patients (ALL and AML) and controls were assessed for the level of five cytokines (IL-2, IL-4, IL-10, IL-17A and IFN-γ) by means of an ELISA method using commercially available kits (PeproTech; UK).

2.9. Statistical analysis

The level of cytokines was given as mean ± standard deviation (SD), and differences between means were assessed by analysis of variance (ANOVA) followed by Duncan test, using the statistical package SPSS version 13.0. The difference was considered significant when the probability was equal or less than 0.05, 0.01 or 0.001.

3. Results

3.1. Total flavonoids

The extract was found to contain 115.73 μg/ml flavonoids. Such finding is in a good agreement with a previous study (Çirak, Radušienė, Janulis, Ivanauskas, & Çamaş, 2011), in which it was demonstrated that *H. triquetrifolium* grown in Turkey is a rich source of flavonoids, such as rutin, hyperoside, apigenin-7-*O*-glucoside, kaempferol, quercitrin, quercetin and amentoflavone.

3.2. IL-2

Serum level of IL-2 showed a significant increase in AML patients (28.2 ± 3.9 pg/ml) compared to ALL patients (23.7 ± 1.9 pg/ml) or controls (10.2 ± 3.2 pg/ml), and the difference was also significant between ALL patients and controls. In culture supernatant, the cells of patients and controls responded differently to the type of treatment. The PHA was significantly able to increase IL-2 level in supernatant of ALL and AML cells (10.4 ± 3.8 and 10.3 ± 3.3 pg/ml, respectively), compared to the corresponding means in untreated cultures (7.4 ± 1.7 and 5.7 ± 2.8 pg/ml, respectively) or PHA-treated control cells (5.9 ± 3.4 pg/ml). Combining PHA and *H. triquetrifolium* methanolic extract contributed to a significant increase in IL-2 level especially at the second concentration of the extract (1.791 μg/ml) in ALL patients (36.4 ± 1.8 pg/ml) compared to controls (11.5 ± 1.5 pg/ml) or AML patients (25.3 ± 1.9 pg/ml). The extract alone and at the second concentration was also effective in inducing the cultured cells of ALL and AML patients to produce a significantly higher level of IL-2 (30.8 ± 1.1 and 21.1 ± 2.6 pg/ml, respectively) compared to controls (21.1 ± 2.6 pg/ml) (Table 1).

3.3. IL-4

Serum level of IL-4 showed a significant decrease in ALL and AML patients (1.5 ± 0.7 and 3.7 ± 2.5 pg/ml, respectively) compared to controls (6.2 ± 1.6 pg/ml). With respect to cultures, the level of IL-4 showed no significant difference between ALL, AML and controls, and the same observation was made when the comparison was made between the different types of treatments for each group of subjects, in which the level of IL-4 was approximated in the six types of cultures that were set in ALL and AML patients or controls (Table 2).

3.4. IL-10

Sera of ALL and AML patients shared a significant increased serum level of IL-10 (38.3 ± 10.4 and 32.9 ± 10.1 pg/ml, respectively) compared to controls (28.1 ± 4.3 pg/ml), while the response of their cultured cells was subjected to the type of treatment. The cells of ALL, control and AML subjects responded similarly to PHA treatment (18.4 ± 8.4, 17.8 ± 5.1 and 17.2 ± 5.2 pg/ml, respectively), while combining PHA and *H. triquetrifolium* methanolic extract revealed some differences in a dose-dependent manner. The highest IL-10 level was observed in supernatants of control (38.9 ± 5.8 pg/ml) and ALL

Table 2. Effect of *H. triquetrifolium* methanolic extract on IL-4 production from cultured mononuclear cells of ALL and AML patients.

Culture number	Plant extract concentration (μg/ml)	IL-4 level Mean ± SD (pg/ml)			p Value* ≤
		ALL (No. = 8)	Controls (No. = 7)	AML (No. = 8)	
I (Untreated)	–	3.3 ± 1.5^{A}	3.5 ± 1.7^{A}	3.1 ± 1.6^{A}	NS
II (PHA only)	–	4.7 ± 1.7^{A}	5.6 ± 1.4^{A}	6.5 ± 3.3^{A}	NS
III (PHA + Extract)	1.146	3.9 ± 1.6^{A}	3.6 ± 1.3^{A}	4.5 ± 1.4^{A}	NS
IV (PHA + Extract)	1.791	3.6 ± 0.7^{A}	4.5 ± 1.5^{A}	4.4 ± 1.8^{A}	NS
V (Extract only)	1.146	4.2 ± 0.9^{A}	3.6 ± 2.3^{A}	4.7 ± 1.3^{A}	NS
VI (Extract only)	1.791	3.7 ± 1.9^{A}	4.9 ± 1.5^{A}	4.4 ± 0.8^{A}	NS
Serum		1.5 ± 0.4^{B}	6.2 ± 2.6^{A}	3.7 ± 2.5^{A}	0.001

ALL: acute lymphoblastic leukemia; AML: acute myelogenous leukemia; NS: not significant; PHA: phytohemagglutinin; SD: standard deviation; Different superscript letters represent a significant difference ($p \leq 0.05$) between means of columns (cultures I–VI and serum).
*Probability of difference between means of ALL, controls and AML.

Table 3. Effect of *H. triquetrifolium* methanolic extract on IL-10 production from cultured mononuclear cells of ALL and AML patients.

Culture number	Plant extract concentration (μg/ml)	IL-10 level Mean ± SD (pg/ml)			p Value* ≤
		ALL (No. = 8)	Controls (No. = 7)	AML (No. = 8)	
I (Untreated)	–	3.2 ± 0.9^{D}	2.6 ± 0.7^{D}	2.7 ± 0.8^{C}	NS
II (PHA only)	–	18.4 ± 8.4^{C}	17.8 ± 5.1^{C}	17.2 ± 5.2^{B}	NS
III (PHA + Extract)	1.146	29.1 ± 5.4^{AB}	29.3 ± 3.1^{B}	15.1 ± 4.1^{B}	0.001
IV (PHA + Extract)	1.791	34.1 ± 5.7^{A}	38.9 ± 5.8^{A}	23.6 ± 8.6^{B}	0.001
V (Extract only)	1.146	27.6 ± 6.9^{B}	26.4 ± 4.4^{B}	20.4 ± 8.1^{B}	NS
VI (Extract only)	1.791	33.7 ± 2.8^{A}	32.5 ± 8.3^{AB}	22.9 ± 9.2^{B}	0.05
Serum		38.3 ± 10.4^{A}	28.1 ± 4.3^{B}	32.9 ± 10.1^{A}	NS

ALL: acute lymphoblastic leukemia; AML: acute myelogenous leukemia; NS: not significant; PHA: phytohemagglutinin; SD: standard deviation; Different superscript letters represent a significant difference ($p \leq 0.05$) between means of columns (cultures I–VI and serum).
*Probability of difference between means of ALL, controls and AML.

(34.1 ± 5.7 pg/ml) cells at the concentration 1.791 μg/ml, while the supernatant of AML cells showed a less level (23.6 ± 8.6 pg/ml) and the difference was significant. Treating cultured cells with the plant extract induced a similar increased production of IL-10, but it did not reach the synergistic effects of PHA and the plant extract (Table 3).

3.5. IL-17A

The sera of ALL and AML patients showed a significant increased level of IL-17A as compared to controls (22.6 ± 6.5 and 27.4 ± 5.3, respectively *vs.* 3.2 ± 1.4 pg/ml). The cultured cells of AML and controls were better than ALL cells in their production of IL-17A after the different treatments (PHA only, PHA + plant extract or plant extract only). The highest production was observed in PHA + plant extract cultures (culture IV) of AML and control subjects (41.1 ± 9.8 and 41.4 ± 6.2 pg/ml, respectively), which were significantly higher than the corresponding culture of ALL patients (23.7 ± 6.8 pg/ml) (Table 4).

3.6. IFN-γ

The sera of ALL patients showed a significant increased level of IFN-γ (70.1 ± 5.5 pg/ml) compared to control (17.6 ± 6.5 pg/ml) or AML (28.1 ± 4.6 pg/ml) sera, but AML patients also manifested a significant increased level compared to controls. The best response of cultured cells in the production of IFN-γ was observed in supernatants of cultures treated with PHA + plant extract at the concentration 1.791μg/ml in ALL and AML patients, and the difference was significant compared to the corresponding supernatants of controls (109.8 ± 18.5 and 86.1 ± 7.1, respectively *vs.* 33.6 ± 8.4 pg/ml) (Table 5).

4. Discussion

The presented results strongly suggest that *H. triquetrifolium* methanolic extract impacted the cells of ALL and AML patients, as well as controls to produce important cytokines that have a role in enhancing cell-mediated immune response (IL-2 and IFN-γ), regulating immune response (IL-10) and mediating anti-inflammatory response (IL-17A); all of which may have a role in controlling the malignant transformation of cells or their progression and produced by important T cells; T helper (Th)1, Treg and Th17 (Levine, 2013; Oppenheim, 2013). In this regard, various medicinal plant-derived factors have been reported to regulate the production of cytokines; for instance, flavonoids (Cuevas, Saavedra, Salazar, & Abdalla, 2013). In fact, the chemical analysis of methanolic extract revealed that *H. triquetrifolium* is a rich source of flavonoids, which were observed at a concentration of 115.73 μg/ml. It has also been augmented that flavonoids that were extracted from different plant sources (e.g. grape seeds, green tea and strawberry) exerted significant immunomodulatory effects by modulating Th1 and Th2-derived

Table 4. Effect of *H. triquetrifolium* methanolic extract on IL-17A production from cultured mononuclear cells of ALL and AML patients.

		IL-17A level Mean ± SD (pg/ml)			
Culture number	Plant extract concentration (μg/ml)	ALL (No. = 8)	Controls (No. = 7)	AML (No. = 8)	p Value* ≤
I (Untreated)	–	1.5 ± 0.4^{C}	2.3 ± 1.3^{C}	2.7 ± 1.2^{C}	NS
II (PHA only)	–	24.9 ± 7.6^{A}	26.2 ± 7.9^{B}	36.9 ± 6.9^{A}	0.01
III (PHA + Extract)	1.146	21.6 ± 5.1^{A}	29.5 ± 7.3^{B}	34.7 ± 5.5^{A}	0.001
IV (PHA + Extract)	1.791	23.7 ± 6.8^{A}	41.4 ± 6.2^{A}	41.1 ± 9.8^{A}	0.001
V (Extract only)	1.146	11.8 ± 5.7^{B}	24.9 ± 4.9^{B}	24.1 ± 3.8^{B}	0.001
VI (Extract only)	1.791	16.3 ± 7.4^{AB}	26.6 ± 3.6^{B}	26.2 ± 3.4^{B}	0.001
Serum		22.6 ± 6.5^{A}	3.2 ± 1.4^{C}	27.4 ± 5.3^{B}	0.001

ALL: acute lymphoblastic leukemia; AML: acute myelogenous leukemia; NS: not significant; PHA: phytohemagglutinin; SD: standard deviation; Different superscript letters represent a significant difference ($p \leq 0.05$) between means of columns (cultures I–VI and serum).
*Probability of difference between means of ALL, controls and AML.

Table 5. Effect of *H. triquetrifolium* methanolic extract on IFN-γ production from cultured mononuclear cells of ALL and AML patients.

		INF-γ level Mean ± SD (pg/ml)			
Culture number	Plant extract concentration (μg/ml)	ALL (No. = 8)	Controls (No. = 7)	AML (No. = 8)	p Value* ≤
I (Untreated)	–	8.4 ± 1.8^{D}	3.3 ± 1.3^{C}	6.7 ± 3.8^{E}	0.01
II (PHA only)	–	40.4 ± 8.9^{C}	13.1 ± 2.5^{B}	12.5 ± 3.8^{D}	0.001
III (PHA + Extract)	1.146	70.9 ± 14.1^{B}	32.1 ± 8.5^{A}	68.6 ± 13.6^{B}	0.001
IV (PHA + Extract)	1.791	109.8 ± 18.5^{A}	33.6 ± 8.4^{A}	86.1 ± 7.1^{A}	0.001
V (Extract only)	1.146	71.5 ± 7.5^{B}	34.2 ± 12.6^{A}	61.9 ± 8.1^{B}	0.001
VI (Extract only)	1.791	77.5 ± 8.8^{B}	33.1 ± 11.1^{A}	61.6 ± 6.5^{B}	0.001
Serum		70.1 ± 5.5^{B}	17.6 ± 6.5^{B}	28.1 ± 4.6^{C}	0.001

ALL: acute lymphoblastic leukemia; AML: acute myelogenous leukemia; PHA: phytohemagglutinin; SD: standard deviation; Different superscript letters represent a significant difference ($p \leq 0.05$) between means of columns (cultures I–VI and serum).
*Probability of difference between means of ALL, controls and AML.

cytokines and other immune cells (Leyva-López, Gutierrez-Grijalva, Ambriz-Perez, & Basilio Heredia, 2016; Nair et al., 2002; Peluso, Miglio, Morabito, Ioannone, & Serafini, 2015).

One of these cytokines is IL-2, which showed an increased level in culture supernatants of plant extract-treated cells. CD4 + T cells are a major source of IL-2, and the increased level of IL-2 might be due to the richness of *H. triquetrifolium* extract in flavonoids that might have enhanced these cells to produce IL-2 (Kunishiro, Tai, & Yamamoto, 2001; Pae & Wu, 2013). It has been demonstrated further that flavonoids can have a wide range of immunomodulatory effects by affecting CD4 + T cells to increase their capacity in IL-2 production both *in vitro* and *in vivo* (Maghraby et al., 2010).

IL-10 was a further target for *H. triquetrifolium* methanolic extract, which exerted stimulating effects on cells to produce IL-10. Again, flavonoids (quercetin) have been demonstrated to have a positive effect on cells to secret IL-10 (Wang et al., 2014). In this context, it has been discussed that flavonoids has immunomodulatory and anti-inflammatory properties and the stimulated activities of numerous cell types (lymphocytes and macrophages) can be influenced by particular flavonoids to produce certain cytokines (Peluso et al., 2015).

The modulation of immune response to combat diseases has long been a topic of interest. Macrophages are the first line of defences in the innate immunity against microbial infection. They engulf and kill microorganisms and present antigens that elicit adaptive immune responses (Farhadi, Mohammadi-Motlagh, Seyfi, & Mostafaie, 2014). Macrophages secrete cytokines, such as interleukins, TNF-α and IFNs, as well as inflammatory mediators, such as nitric oxide (Herbst, Schaible, & Schneider, 2011). The effects of *H. triquetrifolium* methanolic extract on inducing macrophages to release immunomodulatory cytokines, such as IFN-γ, were possibly determined. The results showed that the extract could modulate immunity by inducing macrophages to intracellular expression of IFN-γ. IFN-γ also stimulates macrophages and is involved in the development of Th1 cells. The cellular effects of IFN-γ include up-regulation of pathogen recognition, antigen processing and presentation, inhibition of viruses, inhibition of cellular proliferation, and modifying apoptosis and immunomodulation (Belguendouz et al., 2011; Lyu & Park, 2005). These findings are important and *H. triquetrifolium* can be considered as a very suitable candidate for modulating macrophage function. *In vitro* and *in vivo* studies demonstrated the regulatory effect of phenolic compounds (e.g. the flavones apigenin and luteolin) on macrophage modulation to secret inflammatory mediators (Soromou et al., 2012; Verbeek, Plomp, Van Tol, & Van Noort, 2004). Accordingly, *H. triquetrifolium* might have exerted an immunostimulatory effect on cultured cells by enhancing the expression of IFN-γ, which is involved in lymphocyte activation, and may indirectly affect the activation of NK cell type by inducing macrophages to secret cytokines, which consequently stimulate cell-mediated immunity (Abood, Fahmi, Abdulla, & Ismail, 2014).

The results also showed that the plant extract modulated IL-17A production in a concentration-dependent manner. It might be reasonable to hypothesize that flavonoids of the plant extract may regulate RORγt (the first transcription factor selectively expressed in Th17 cells), which is regulated by STAT3 (signal transducer and activator of transcription 3). The latter acts as an important mediator in multiple biological processes induced by different cytokines (Ivanov, Zhou, Littman, & Ivanov, 2007; Wei, Laurence, Elias, & O'Shea, 2007).

It was also observed that ALL, AML or control cultured mononuclear cells responded differently to the plant extract in their production of IL-2, IL-10, IL-17A and IFN-γ, but leukemic cells were better than control cells in their response, and this may suggest that these cells were pre-committed to interior immunological effects and the plant extract came to enhance such effects. Some herbal medicines are suggested to alter the activity of immune function through a dynamic regulation of informational molecules such as cytokines. This may offer an explanation for the effects of herbs on function of immune system and other tissue, especially in leukemia, and *H. triquetrifolium* might be one of these herbal medicines that modulate production of cytokines (Novelli et al., 2014; Xiuying et al., 2012; Saad et al., 2011). However, the present results were based on *in vitro* assessments of the plant extract effects, and an *in vivo* assessment may show a different effect. Lee, Hah, and Kim (2012) investigated this possibility in 19 children received Korean red ginseng extract for 12 months after cessation of chemotherapy or stem cell transplantation for leukemia, lymphoma or solid tumor. Their results indicated that serum level of inflammatory cytokines (IL-2, IL-10, IL-12, TNF-α and IFN-γ) showed a rapid decreasing in extract-treated patients compared to untreated controls. Accordingly, the *in vivo* metabolism of plant extract may render it to have different effects on production of cytokines.

5. Conclusions

The *in vitro* immunomodulatory potential of *H. triquetrifolium* methanolic extract is suggested, and the profiles of IL-2, IL-10, IL-17A and IFN-γ in the supernatant of cultured ALL, AML and control cells are in favor of such suggestion. However, further investigations based on *in vivo* evidence are certainly required.

Acknowledgments

The authors appreciate the kind help and cooperation of the medical staff at the Baghdad Teaching Hospital. We are also grateful to Dr. Saman Abdulrahman Ahmad (Department of Field Crops, College of Agriculture Science, University of Sulaymaniyah, Iraq), who supplied the plant powder.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Ali H. Ad'hiah https://orcid.org/0000-0002-2445-2242

References

Abood, W. N., Fahmi, I., Abdulla, M. A., & Ismail, S. (2014). Immunomodulatory effect of an isolated fraction from *Tinospora crispa* on intracellular expression of INF-γ, IL-6 and IL-8. *BMC Complementary and Alternative Medicine*, *14*, 205. doi:10.1186/1472-6882-14-205

Ad'hiah, A. H. (1990). Immunogenetic studies in selected human diseases. Ph.D. Thesis, Department of Human Genetics, University of Newcastle upon Tyne, England.

Belguendouz, H., Messaoudène, D., Lahmar, K., Ahmedi, L., Medjeber, O., Hartani, D., …, Touil-Boukoffa, C. (2011). Interferon-γ and nitric oxide production during Behçet uveitis: immunomodulatory effect of interleukin-10. *Journal of Interferon & Cytokine Research*, *31*, 643–651. doi:10.1089/jir.2010.0148

Canning, S., Waterman, M., Orsi, N., Ayres, J., Simpson, N., & Dye, L. (2010). The Efficacy of *Hypericum perforatum* (St John's Wort) for the treatment of premenstrual syndrome: a randomized, double-blind, placebo-controlled trial. *CNS Drugs*, *24*, 207–225. doi:10.2165/11530120-000000000-00000

Chiarini, F., Lonetti, A., Evangelisti, C., Buontempo, F., Orsini, E., Evangelisti, C., …, Martelli, A. M. (2015). Advances in understanding the acute lymphoblastic leukemia bone marrow microenvironment: from biology to therapeutic targeting. *Biochimica et Biophysica Acta*, *1863*, 449–463. doi:10.1016/j.bbamcr.2015.08.015

Chouhan, G., Islamuddin, M., Sahal, D., & Afrin, F. (2014). Exploring the role of medicinal plant-based immunomodulators for effective therapy of leishmaniasis. *Frontiers in Immunology*, *5*, 193. doi:10.3389/fimmu.2014.00193

Çirak, C., Radušienė, J., Janulis, V., Ivanauskas, L., & Çamaş, N. (2011). Phenolic constituents of *Hypericum triquetrifolium* Turra (Guttiferae) growing in Turkey: variation among populations and plant parts. *Turkish Journal of Biology*, *35*, 449–456. doi:10.3906/biy-1002-36

Conforti, F., Loizzo, M. R., Statti, A. G., & Menichini, F. (2007). Cytotoxic activity of antioxidant constituents from *Hypericum triquetrifolium* Turra. *Natural Products Research*, *21*, 42–46. doi:10.1080/14786410500356243

Cuevas, A., Saavedra, N., Salazar, L. A., & Abdalla, D. S. P. (2013). Modulation of immune function by polyphenols: possible contribution of epigenetic factors. *Nutrients*, *5*, 2314–2332. doi:10.3390/nu5072314

Farhadi, L., Mohammadi-Motlagh, H. -R., Seyfi, P., & Mostafaie, A. (2014). Low concentrations of flavonoid - rich fraction of shallot extract induce delayed - type hypersensitivity and TH1 cytokine IFNγ expression in BALB/c mice. *International Journal of Molecular and Cellular Medicine*, *3*, 16–25.

Fu, W., Chen, J., Cai, Y., Lei, Y., Chen, L., Pei, L., …, Ruan, J. (2010). Antioxidant, free radical scavenging, anti-inflammatory and hepatoprotective potential of the extract from *Parathelypteris nipponica* (Franch. et Sav.) Ching. *Journal of Ethnopharmacology*, *130*, 521–528. doi:10.1016/j.jep.2010.05.039

Herbst, S., Schaible, U. E., & Schneider, B. E. (2011). Interferon gamma activated macrophages kill mycobacteria by nitric oxide induced apoptosis. *PLoS One*, *6*, e19105. doi:10.1371/journal.pone.0019105

Hu, Z. P., Yang, X. X., Chan, S. Y., Xu, A. L., Duan, W., Zhu, Y. Z., …, Zhou, S. F. (2006). St. John's wort attenuates irinotecan-induced diarrhea via down-regulation of intestinal pro-inflammatory cytokines and inhibition of intestinal epithelial apoptosis. *Toxicology and Applied Pharmacology*, *216*, 225–237. doi:10.1016/j.taap.2006.05.020

Ivanov, I. I., Zhou, L., & Littman, D. R. (2007). Transcriptional regulation of Th17 cell differentiation. *Seminars in Immunology*, *19*, 409–417. doi:10.1016/j.smim.2007.10.011

Kunishiro, K., Tai, A., & Yamamoto, I. (2001). Effects of rooibos tea extract on antigen-specific antibody production and cytokine generation *in vitro* and *in vivo*. *Bioscience, Biotechnology, and Biochemistry*, *65*, 2137–2145. doi:10.1271/bbb.65.2137

Lee, J. M., Hah, J. O., & Kim, H. S. (2012). The effect of red ginseng extract on inflammatory cytokines after chemotherapy in children. *Journal of Ginseng Research*, *36*, 383–390. doi:10.5142/jgr.2012.36.4.383

Levine, S. J. (2013). Cytokines. In *Encyclopedia of biological chemistry* (2nd ed., pp. 613–621). USA: Elsevier Inc. doi.10.1016/B978-0-12-378630-2.09990-4

Leyva-López, N., Gutierrez-Grijalva, E. P., Ambriz-Perez, D. L., & Basilio Heredia, J. (2016). Flavonoids as cytokine modulators: apossible therapy for inflammation-related diseases. *International Journal of Molecular Sciences*, *17*, pii: E921. doi:10.3390/ijms17060921

Lyu, S. -Y., & Park, W. -B. (2005). Production of cytokine and NO by RAW 264.7 macrophages and PBMC *in vitro* incubation with flavonoids. *Archives of Pharmacal Research*, *28*, 573–581. doi:10.1007/BF02977761

Maghraby, A. S., Shalaby, N., Abd-Alla, H. I., Ahmed, S. A., Khaled, H. M., & Bahgat, M. M. (2010). Immunostimulatory effects of extract of *Pulicaria crispa* before and after *Schistosoma mansoni* infection. *Acta Poloniae Pharmaceutica Drug Research*, *67*, 75–79.

Nair, N., Mahajan, S., Chawda, R., Kandaswami, C., Shanahan, T. C., & Schwartz, S. A., 2002. Grape seed extract activates Th1 cells in vitro. *Clinical and Vaccine Immunology*, *9*, 470–476. doi:10.1128/CDLI.9.2.470-476.2002

Novelli, M., Beffy, P., Menegazzi, M., De Tata, V., Martino, L., Sgarbossa, A., …, Masiello, P. (2014). St. John's wort extract and hyperforin protect rat and human pancreatic islets against cytokine toxicity. *Acta Diabetologica*, *51*, 113–121. doi:10.1007/s00592-013-0518-2

Oppenheim, J. J. (2013). Cytokines, their receptors and signals. In *The autoimmune diseases* (5th ed., pp. 229–241). USA: Elsevier Inc. doi:10.1016/B978-0-12-384929-8.00083-6

Pae, M., & Wu, D. (2013). Immunomodulating effects of epigallocatechin-3-gallate from green tea: mechanisms and applications. *Food & Function*, *4*, 1287–1303. doi:10.1039/c3fo60076a

Peluso, I., Miglio, C., Morabito, G., Ioannone, F., & Serafini, M. (2015). Flavonoids and immune function in human: A systematic review. *Critical Reviews in Food Science & Nutrition*, *55*, 383–395. doi:10.1080/10408398.2012.656770

Xiuying, P., Liang, J., Shang, R., Zhou, L., Wang, X., & Li, Y. (2012). Therapeutic efficacy of *Hypericum perforatum* L. extract for mice infected with an influenza A virus. *Canadian Journal of Physiology and Pharmacology*, *90*, 123–130. doi:10.1139/y11-111

Rouis, Z., Abid, N., Koudja, S., Yangui, T., Elaissi, A., Cioni, P.L., …, Aouni, M. (2013). Evaluation of the cytotoxic effect and antibacterial, antifungal, and antiviral activities of *Hypericum triquetrifolium* Turra essential oils from Tunisia. *BMC Complementary and Alternative Medicine*, *13*, 24. doi:10.1186/1472-6882-13-24

Saad, B., Abouatta, B. S., Basha, W., Hmade, A., Kmail, A., Khasib, S., & Said, O. (2011). *Hypericum triquetrifolium* – derived factors downregulate the production levels of LPS-induced nitric oxide and tumor necrosis factor-α in THP-1 cells. *Evidence-based Complementary and Alternative Medicine*, *2011*. doi:10.1093/ecam/nen056

Saad, B., Azaizeh, H., & Said, O. (2005). Tradition and perspectives of Arab herbal medicine: A review. *Evidence-based Complementary and Alternative Medicine*, *2*, 475–479. doi:10.1093/ecam/neh133

Sakanaka, S., Tachibana, Y., & Okada, Y. (2005). Preparation and antioxidant properties of extracts of Japanese persimmon leaf tea (kakinoha-cha). *Food Chemistry*, *89*, 569–575. doi:10.1016/j.foodchem.2004.03.013

Saxena, A., Yadav, D., Maurya, A. K., Kumar, A., Mohanty, S., Gupta, M. M., …, Bawankule, D. U. (2016). Diarylheptanoids from *Alnus nepalensis* attenuates LPS-induced inflammation in macrophages and endotoxic shock in mice. *International Immunopharmacology*, *30*, 129–136. doi:10.1016/j.intimp.2015.12.002

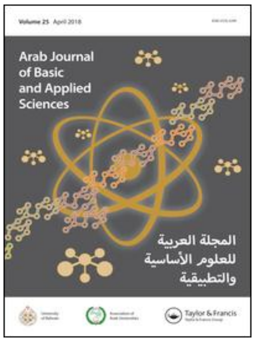
Soromou, L. W., Zhang, Z., Li, R., Chen, N., Guo, W., Huo, M., …, Deng, X. (2012). Regulation of inflammatory cytokines in lipopolysaccharide-stimulated RAW 264.7 murine macrophage by 7-O-methyl-naringenin. *Molecules*, *17*, 3574–3585. doi:10.3390/molecules17033574

Van Etten, R. A. (2007). Aberrant cytokine signaling in leukemia. *Oncogene*, *26*, 6738–6749. doi:10.1038/sj.onc.1210758

Verbeek, R., Plomp, A. C., Van Tol, E. A. F., & Van Noort, J. M. (2004). The flavones luteolin and apigenin inhibit *in vitro* antigen-specific proliferation and interferon-gamma production by murine and human autoimmune T cells. *Biochemical Pharmacology*, *68*, 621–629. doi:10.1016/j.bcp.2004.05.012

Wang, L., Chen, J., Wang, B., Wu, D., Li, H., Lu, H., …, Chai, Y. (2014). Protective effect of quercetin on lipopolysaccharide-induced acute lung injury in mice by inhibiting inflammatory cell influx. *Experimental Biology and Medicine (Maywood)*, *239*, 1653–1662. doi:10.1177/1535370214537743

Wei, L., Laurence, A., Elias, K. M., & O'Shea, J. J. (2007). IL-21 is produced by Th17 cells and drives IL-17 production in a STAT3-dependent manner. *Journal of Biological Chemistry*, *282*, 34605–34610. doi:10.1074/jbc.M705100200



Arab Journal of Basic and Applied Sciences

ISSN: (Print) 2576-5299 (Online) Journal homepage: http://www.tandfonline.com/loi/tabs20

Daftardar-Jafari method for solving nonlinear thin film flow problem

Majeed Ahmed AL-Jawary, Ghassan Hasan Radhi & Jure Ravnik

ARAB JOURNAL OF BASIC AND APPLIED SCIENCES
2018, VOL. 25, NO. 1, 20–27
https://doi.org/10.1080/25765299.2018.1449345

University of Bahrain
Taylor & Francis
Taylor & Francis Group

ORIGINAL ARTICLE
OPEN ACCESS
Check for updates

Daftardar-Jafari method for solving nonlinear thin film flow problem

Majeed Ahmed AL-Jawary[a], Ghassan Hasan Radhi[a] and Jure Ravnik[b]

[a]Department of Mathematics, University of Baghdad, College of Education for Pure Science (Ibn AL-Haitham), Baghdad, Iraq;
[b]Faculty of Mechanical Engineering, University of Maribor, Maribor, Slovenia

ABSTRACT
The aim of this paper is to develop the Daftardar-Jafari iterative method (DJM) for a mathematical model that represents the nonlinear thin film flow of a non-Newtonian third-grade fluid on a moving belt with the aim to obtain an approximate solution of high accuracy. When applying the DJM there is no need to resort to any additional techniques such as evaluating Adomian's polynomials as in the Adomian decomposition method (ADM) or such as using Lagrange multipliers in the variational iteration method (VIM). The accuracy of our results is numerically verified by evaluating the functions of the error remainder and the maximal error remainders. In addition, these results are analyzed by comparing the accuracy of the DJM solutions with those of the fourth order Runge-Kutta method (RKM), ADM and VIM at the same parameter values. All the evaluations have been successfully performed in an iterative way by using the symbolic manipulator *Mathematica*®.

ARTICLE HISTORY
Received 4 June 2017
Accepted 2 December 2017

KEYWORDS
Daftardar-Jafari iterative method; Runge–Kutta method; thin film flow; nonlinear boundary value problem; approximate solution

1. Introduction

In recent decades, the use of numerical methods has become a standard way to solve and evaluate different types of complex nonlinear problems. In this paper, we have proposed and developed an alternative approach – using iterative methods to find a solution with a high degree of accuracy. The iterative procedure leads to a series, which can be summed up to find an analytical formula, or it can form a suitable approximation. The error of the approximation can be controlled by properly truncating the series.

The subject of this study is about non-Newtonian fluids. Unlike Newtonian fluids, where the shear stress is linearly proportional to strain rate, the non-Newtonian fluid exhibit behaviour that is more complex. Examples of non-Newtonian fluids are salt solutions and molten polymers. Non-Newtonian fluids have been studied extensively in the last decades (Rajagopal, 1983) and are currently still a focus of many researchers (Bhatti, Zeeshan, & Ellahi, 2016; Rashidi, Bagheri, Momoniat, & Freidoonimehr, 2017; Ravnik & Skerget, 2015; Sheikholeslami & Zeeshan, 2017; Zeeshan & Atlas, 2017; Zeeshan et al., 2016).

Several iterative methods have been previously proposed for finding solutions of initial or boundary value problems. The most common are: the Adomian decomposition method (ADM) (Adomian, 1994; Siddiqui, Hameed, Siddiqui, & Ghori, 2010), the variational iteration method (VIM) (He, 1999b), the homotopy analysis method (HAM) (Liao, 2004), the homotopy perturbation method (HPM) (He, 1999a, 2000) and the differential transform method (DTM) (Bildik, Konuralp, Bek, & Kucukarslan, 2006; Zhou, 1986), etc.

In this paper, we implemented the Daftardar-Jafari method (DJM) (Daftardar-Gejji & Jafari, 2006) to solve the thin film flow of a third grade fluid on a moving belt. Our aim was to find an approximate solution without using any restricted assumptions. The DJM has been introduced for the first time by Varsha Daftardar-Gejji and Hossein Jafari in 2006. This iterative method has been successfully used to solve many kinds of problems. For instance; the application of DJM for solving different kinds of partial differential equations (Bhalekar & Daftardar-Gejji, 2008, 2012; Daftardar-Gejji & Bhalekar, 2010), solving the Laplace equation (Yaseen et al., 2013), solving the Volterra integro-differential equations with some applications for the Lane-Emden equations of the first kind (AL-Jawary & AL-Qaissy, 2015), solving the Fokker-Planck equation (AL-Jawary, 2016), Duffing equations (Al-Jawary & Al-Razaq, 2016) and calculating the steady-state concentrations of carbon dioxide absorbed into phenyl glycidyl ether solutions (Al-Jawary & Raham, 2016), and others. The thin film flow problem has been solved previously by the ADM and VIM (Siddiqui, Farooq, Haroon, & Babcock, 2012a), the semi-analytical iterative method by

CONTACT Majeed Ahmed AL-Jawary Majeed.a.w@ihcoedu.uobaghdad.edu.iq Department of Mathematics, University of Baghdad, College of Education for Pure Science (Ibn AL-Haitham), Baghdad, Iraq

Temimi and Ansari (TAM) (AL-Jawary, 2017) and other known iterative methods (Gul, Islam, Shah, Khan, & Shafie, 2014; Mabood, 2014; Mabood & Pochai, 2015; Moosavi, Momeni, Tavangar, Mohammadyari, & Rahimi-Esbo, 2016; Nemati, Ghanbarpour, Hajibabayi, & Hemmatnezhad, 2009; Sajid & Hayat, 2008; Shah, Pandya, & Shah, 2016; Siddiqui, Farooq, Haroon, Rana, & Babcock, 2012b). The following sections review the application of the DJM to solve the current problem and the validity of this method in finding the appropriate approximate solution.

2. The nonlinear thin film flow problem

In this section, we consider the thin film flow of non-Newtonian fluid on a moving belt (Siddiqui et al., 2012a). The flow is steady, laminar and uniform. The film thickness is also uniform. The following problem is governed by (Siddiqui et al., 2012a):

$$\frac{d^2w}{dx^2}+\frac{6(\beta_2+\beta_3)}{\mu}\left(\frac{dw}{dx}\right)^2\frac{d^2w}{dx^2}-\frac{\delta f}{\mu}=0, \quad (1)$$

$$w(0)=V_0,\ \frac{dw}{dx}=0 \text{ at } x=\gamma, \quad (2)$$

where, w represents the fluid velocity, β_2 and β_3 are the material constants of the third-grade fluid, μ represents the dynamic viscosity, δ is the density, f is the acceleration with respect to gravity, γ is the uniform thickness of the film and V_0 is the speed of the belt.

The following dimensionless variables can be introduced as follows:

$$\tilde{x}=\frac{x}{\gamma},\quad \tilde{w}=\frac{w}{V_0},\quad \beta=\frac{(\beta_2+\beta_3)V_0^2}{\mu\gamma^2},\quad m=\frac{\delta f\gamma^2}{\mu V_0}. \quad (3)$$

The dimensionless form of the nonlinear boundary value problem of (1) and (2) with $\sim$ removed is

$$\frac{d^2w}{dx^2}+6\beta\left(\frac{dw}{dx}\right)^2\frac{d^2w}{dx^2}-m=0, \quad (4)$$

$$w(0)=1,\ \frac{dw}{dx}=0 \text{ at } x=1. \quad (5)$$

Since Equation (4) has two boundary conditions and since it is a second order nonlinear ODE it is considered to be a well-posed problem. By integrating Equation (4) twice and by using the boundary conditions given in (5), one can arrive to

$$\frac{dw}{dx}+2\beta\left(\frac{dw}{dx}\right)^3-mx=C, \quad (6)$$

where, C is the integration constant. When using the second condition shown in Equation (5) to calculate the integration constant in (6), the integration constant will be $C=-m$. Thus, the nonlinear system of (4) and (5) can be represented with the following problem:

$$\frac{dw}{dx}+2\beta\left(\frac{dw}{dx}\right)^3-m(x-1)=0,\quad w(0)=1. \quad (7)$$

In the next sections, the basic steps of the DJM will be reviewed and applied to find an approximate solution for the problem presented by Equation (7).

3. The Daftardar-Jafari method

In order to demonstrate the steps of using the DJM; we first begin with considering the following general functional equation (Daftardar-Gejji & Jafari, 2006).

$$w=f+L(w)+N(w), \quad (8)$$

where, L denotes the linear operator, N is the nonlinear operator, f represents a given function and w is the solution for equation (8), which can be written as

$$w=\sum_{i=0}^{\infty}w_i. \quad (9)$$

Now, the following can be defined

$$G_0=N(w_0), \quad (10)$$

$$G_m=N\left(\sum_{i=0}^{m}w_i\right)-N\left(\sum_{i=0}^{m-1}w_i\right), \quad (11)$$

so that $N(w)$ can decomposed as

$$N\left(\sum_{i=0}^{\infty}w_i\right)=\underbrace{N(w_0)}_{G_0}+\underbrace{[N(w_0+w_1)-N(w_0)]}_{G_1}+\underbrace{[N(w_0+w_1+w_2)-N(w_0+w_1)]}_{G_2}+\underbrace{[N(w_0+w_1+w_2+w_3)-N(w_0+w_1+w_2)]}_{G_3}+\dots \quad (12)$$

Moreover, the relation is defined with recurrence so that

$$w_0=f, \quad (13)$$

$$w_1=L(w_0)+G_0, \quad (14)$$

$$w_{m+1}=L(w_m)+G_m,\quad m=1,2,\dots. \quad (15)$$

Since L represents a linear operator $\sum_{i=0}^{m}L(w_i)=L(\sum_{i=0}^{m}w_i)$, we may write

$$\sum_{i=1}^{m+1}w_i=\sum_{i=0}^{m}L(w_i)+N\left(\sum_{i=0}^{m}w_i\right)=L\left(\sum_{i=0}^{m}w_i\right)+N\left(\sum_{i=0}^{m}w_i\right),\quad m=1,2,\dots. \quad (16)$$

So that,

$$\sum_{i=0}^{\infty} w_i = f + L\left(\sum_{i=0}^{\infty} w_i\right) + N\left(\sum_{i=0}^{\infty} w_i\right). \quad (17)$$

From the equation above, it is clear that $\sum_{i=0}^{\infty} w_i$ is the solution for Equation (8), where the functions w_i, $i = 0, 1, 2, \ldots$ are obtained by the algorithm (13)–(15). The k-term series solution, which is given by $w = \sum_{i=0}^{k-1} w_i$, represents an approximate solution for Equation (17).

3.1. The convergence of the DJM

In 1922, the fixed point theorem has been proposed by Stefan Banach "1892–1945" (Banach, 1922). This theorem is very important in the field of functional analysis. Let us review it here.

Definition 3.1: (Banach, 1922) Let (X, d) be a metric space and let $N : X \to X$ be a Lipschitz continuous mapping then N is called a contraction mapping, if there exists a constant $0 \le k < 1$ such that $d(N(x), N(y)) \le k.d(x, y)$, for all $x, y \in X$.

Banach fixed point theorem: (Banach, 1922) Let (X, d) be a complete metric space and $N : X \to X$ be a contraction mapping then N admits a unique fixed point x_f in X, i.e. $N(x_f) = x_f$. Also x_f can be found as follows:

Starting with an arbitrary element x_0 in X and then defining a sequence $\{x_n\}$ as $x_n = N(x_{n-1})$, then $x_n \to x_f$.

Theorem 3.1: (Biazar & Ghazvini, 2009) Let X and Y be Banach spaces and $N : X \to Y$ be a contraction nonlinear mapping such that for some constant $0 \le k < 1$

$$\|N(u) - N(u_f)\| \le k\|u - u_f\|, \ \forall \ u, \ u_f \in X,$$

Where, according to the fixed point theorem of Banach, there is a fixed point w such that $N(w) = w$, hence the generated terms by the DJM will regarded as

$w_n = N(w_{n-1}), \lim_{n\to\infty} w_n = w$, and suppose that $w_0 \in B_r(w)$

where $B_r(w) = \{w^* \in X; \|w^* - w\| < r\}$ then we have the following statements:

1. $\|w_n - w\| \le k^n \|w_0 - w\|$,
2. $w_n \in B_r(w)$,
3. $\text{Lim}_{n\to\infty} w_n = w$.

Proof: See (Biazar & Ghazvini, 2009).

In order to analyze the convergence of the DJM for solving the problem (8), we consider two solutions: w_{DJM} and w_{RKM}. The first is the approximate solution, which is obtained by the DJM and the second is a numerical solution, which is obtained by using the Runge Kutta method (RKM) (AL-Jawary, 2017).

Now let $w_{RKM} - w_{DJM} = e$ be the error of the evaluated solutions w_{RKM} and w_{DJM} of (8). Let e satisfy (8) such that

$$e = f + L(e) + N(e). \quad (18)$$

Then the recurrence relation in (13)–(15) will take the following form

$$e_0 = f, \quad (19)$$

$$e_1 = L(e_0) + N(e_0), \quad (20)$$

$$e_{m+1} = L(e_m) + N\left(\sum_{i=0}^{m} e_i\right) - N\left(\sum_{i=0}^{m-1} e_i\right), m = 1, 2, \ldots. \quad (21)$$

According to the nonlinear contraction mapping Theorem 3.1; if $\|w_n - w\| \le k^n \|w_0 - w\|$, $0 \le k < 1$ then

$$\begin{aligned} e_0 &= f, \\ \|e_1\| &= \|N(e_0)\| \le k\|e_0\|, \\ \|e_2\| &= \|N(e_0 + e_1) - N(e_0)\| \\ &= \|N(e_1)\| \le k\|e_1\| \le k(k\|e_0\|) = k^2\|e_0\|, \end{aligned}$$

therefore $\|e_2\| \le k^2\|e_0\|$.

$$\begin{aligned} \|e_3\| &= \|N(e_0 + e_1 + e_2) - N(e_0 + e_1)\| \\ &= \|N(e_2)\| \le k\|e_2\| \le k(k\|e_1\|) \le k(k(k\|e_0\|)) \\ &= k^3\|e_0\|, \end{aligned}$$

therefore $\|e_3\| \le k^3\|e_0\|$

In general, we have $\|e_{n+1}\| \le k^{n+1}\|e_0\|$.

So that as $n \to \infty$ the error $e_{n+1} \to 0$ and that proves the convergence of the DJM for the general functional Equation (8). Please refer to (Bhalekar & Daftardar-Gejji, 2011; Hemeda, 2013) for more details.

4. Solving the governing problem by the DJM

In order to use the DJM to find an approximate solution for the problem (7), we have rewritten below this equation in the following way

$$\frac{dw}{dx} = m(x-1) - 2\beta\left(\frac{dw}{dx}\right)^3, \ \ w(0) = 1. \quad (22)$$

By integrating (22) and using the given initial condition, we get

$$w = 1 - mx + m\frac{x^2}{2} - 2\beta \int_0^x \left(\frac{dw}{dt}\right)^3 dt. \quad (23)$$

We have $N(w) = -2\beta \int_0^x \left(\frac{dw}{dt}\right)^3 dt$ and $f = 1 - mx + m\frac{x^2}{2}$. Now, by applying the basic steps of the

DJM, we obtain the following set of approximations

$$w_0 = 1 - mx + m\frac{x^2}{2},$$

$$w_1 = -\frac{1}{2}m^3\left(-1 + (-1+x)^4\right)\beta,$$

$$\begin{aligned} w_2 = {}& \frac{1}{5}m^5(-2+x)x\beta^2\Big(-64m^4x^7\beta^2 + 8m^4x^8\beta^2 + 2m^2x^5\beta(45 - 248m^2\beta) \\ & + m^2x^6\beta(-15 + 232m^2\beta) + 10(3 - 6m^2\beta + 4m^4\beta^2) \\ & - 20x(3 - 9m^2\beta + 8m^4\beta^2) - 40x^3(1 - 9m^2\beta + 16m^4\beta^2) \\ & + 10x^2(7 - 33m^2\beta + 40m^4\beta^2) + 2x^4(5 - 120m^2\beta + 344m^4\beta^2)\Big), \end{aligned}$$

$$\vdots$$

The series solution $w_{DJM,n}(x) = \sum_{i=0}^{k} w_i$ for Equation (22) can be derived by making the sum of the above components w_i obtained by the DJM. For ease and brevity, we mention the following series:

$$\begin{aligned} w_{DJM,2}(x) = {}& \sum_{i=0}^{2} w_i = 1 + m\left(-1 + \frac{x}{2}\right)x \\ & - 2\beta\Big(-m^3x + \frac{3m^3x^2}{2} - m^3x^3 + \frac{m^3x^4}{4} + 6m^5x\beta \\ & - 15m^5x^2\beta + 20m^5x^3\beta - 15m^5x^4\beta + 6m^5x^5\beta \\ & - m^5x^6\beta - 12m^7x\beta^2 + 42m^7x^2\beta^2 - 84m^7x^3\beta^2 \\ & + 105m^7x^4\beta^2 - 84m^7x^5\beta^2 + 42m^7x^6\beta^2 - 12m^7x^7\beta^2 \\ & + \frac{3}{2}m^7x^8\beta^2 + 8m^9x\beta^3 - 36m^9x^2\beta^3 + 96m^9x^3\beta^3 \\ & - 168m^9x^4\beta^3 + \frac{1008}{5}m^9x^5\beta^3 - 168m^9x^6\beta^3 \\ & + 96m^9x^7\beta^3 - 36m^9x^8\beta^3 + 8m^9x^9\beta^3 - \frac{4}{5}m^9x^{10}\beta^3\Big). \end{aligned} \tag{24}$$

In the next subsection, we present the difference between the approximate solution of the DJM and the three standard variational iteration algorithms (He, Wu, & Austin, 2010; He, 2012).

4.1. The VIM algorithms

As in (He et al., 2010; He, 2012), the following form of nonlinear equation has been considered

$$Lw + Nw = 0, \tag{25}$$

where, L and N are the linear and nonlinear operators of this equation, respectively.

According to the VIM (He, 1999b), three variational iterative algorithms can be applied for solving the current nonlinear problem (7) (He, 2012).

Variational iteration algorithm-I:

$$w_{n+1}(x) = w_n(x) + \int_{x_0}^{x} \lambda\{Lw_n(t) + Nw_n(t)\}dt. \tag{26}$$

Variational iteration algorithm-II:

$$w_{n+1}(x) = w_0(x) + \int_{x_0}^{x} \lambda Nw_n(t)dt. \tag{27}$$

Variational iteration algorithm-III:

$$w_{n+2}(x) = w_{n+1}(x) + \int_{x_0}^{x} \lambda\{Nw_{n+1}(t) - Nw_n(t)\}dt, \tag{28}$$

Where, the Lagrange multiplayer λ has been systematically explained in (He, 1999b). In general, when applying the VIM for solving (7), one selects $\lambda = -1$ and the nonlinear operator is $Nw(x) = 2\beta\left(\frac{dw}{dx}\right)^3$. The employed functional when applying the variational iterative algorithm-I for solving (7) finally takes in the following form:

$$\begin{aligned} w_{1,n+1}(x) = {}& w_{1,n}(x) \\ & - \int_0^x \left(\frac{dw_{1,n}}{dt} + 2\beta\left(\frac{dw_{1,n}}{dt}\right)^3 - m(t-1)\right)dt, \end{aligned} \tag{29}$$

where, $w_{1,0}(x) = 1$ and the other iterations are:

$$w_{1,1}(x) = 1 - mx + \frac{mx^2}{2},$$

$$w_{1,2}(x) = 1 - mx + \frac{mx^2}{2} - \frac{1}{2}m^3\left(-1 + (-1+x)^4\right)\beta,$$

$$\begin{aligned} w_{1,3}(x) = {}& 1 - mx + \frac{mx^2}{2} - \frac{1}{2}m^3\left(-1 + (-1+x)^4\right)\beta \\ & - 12m^5x\beta^2 + 30m^5x^2\beta^2 - 40m^5x^3\beta^2 + 30m^5x^4\beta^2 \\ & - 12m^5x^5\beta^2 + 2m^5x^6\beta^2 + 24m^7x\beta^3 - 84m^7x^2\beta^3 \\ & + 168m^7x^3\beta^3 - 210m^7x^4\beta^3 + 168m^7x^5\beta^3 \\ & - 84m^7x^6\beta^3 + 24m^7x^7\beta^3 - 3m^7x^8\beta^3 - 16m^9x\beta^4 \\ & + 72m^9x^2\beta^4 - 192m^9x^3\beta^4 + 336m^9x^4\beta^4 \\ & - \frac{2016}{5}m^9x^5\beta^4 + 336m^9x^6\beta^4 - 192m^9x^7\beta^4 \\ & + 72m^9x^8\beta^4 - 16m^9x^9\beta^4 + \frac{8}{5}m^9x^{10}\beta^4, \end{aligned}$$

$$\vdots$$

and so on. When applying the variational iterative algorithm-II for solving (7) the form of the employed functional reads as:

$$w_{2,n+1}(x) = w_{2,0}(x) - \int_0^x \left(2\beta\left(\frac{dw_{2,n}}{dt}\right)^3 - m(t-1)\right)dt, \tag{30}$$

and the final form of the functional used in the application of algorithm-III to solve (7) is

$$\begin{aligned} w_{3,n+2}(x) = {}& w_{3,n+1}(x) \\ & - \int_0^x \left(2\beta\left(\frac{dw_{3,n+1}}{dt}\right)^3 - 2\beta\left(\frac{dw_{3,n}}{dt}\right)^3 - m(t-1)\right)dt. \end{aligned} \tag{31}$$

The approximate terms obtained by Equations (29), (30) and (31) are all the same.

After simplifying both of the DJM series form, i.e. $w_{DJM,n}(x) = \sum_{i=0}^{k} w_i(x)$ and the nth iteration obtained by the VIM $w_{VIM,n}(x)$; we observe that:

$$w_{DJM,n}(x) = w_{VIM,n+1}(x). \tag{32}$$

Consider this example: when making a simplification for $w_{DJM,2}(x)$ mentioned in (24) and $w_{VIM,3}(x)$ we have:

$$\begin{aligned} w_{DJM,2}(x) = w_{VIM,3}(x) = 1 + \frac{1}{2}m(-2+x)x \\ -\frac{1}{2}m^3x(-4+6x-4x^2+x^3)\beta \\ +2m^5x(-6+15x-20x^2+15x^3-6x^4+x^5)\beta^2 \\ -3m^7x(-8+28x-56x^2+70x^3-56x^4+28x^5 \\ -8x^6+x^7)\beta^3+\frac{8}{5}m^9x(-10+45x-120x^2 \\ +210x^3-252x^4+210x^5-120x^6+45x^7 \\ -10x^8+x^9)\beta^4. \end{aligned}$$

It is worth mentioning that the nth iteration $w_{VIM,n}(x)$ represents the approximate solution obtained by applying any of the three standard variational iteration algorithms (29), (30) and (31).

We used Mathematica, the symbolic computation and manipulation software in our calculations. To check the accuracy of this approximate solution, we have suggested the following error remainder function

$$\begin{aligned} ER_n(x) = \frac{d}{dx}\left(\sum_{i=0}^{n} w_i\right) + 2\beta\left(\frac{d}{dx}\left(\sum_{i=0}^{n} w_i\right)\right)^3 \\ -m(x-1) = 0, \end{aligned} \tag{33}$$

with the maximal error remainder parameter

$$MER_n = \max_{0\le x\le 1} |ER_n(x)|, \tag{34}$$

All the terms that involve β and its powers give the contribution for the non-Newtonian fluid. Moreover, when setting $\beta = 0$ in the approximations above, we can retrieve the exact solution for the current problem of the Newtonian viscous fluid.

5. Numerical simulations and results

When inserting the values of β and m in the approximate solution (24) we can get several approximate solutions. We have chosen $\beta = 0.5$ and $m = 0.3$ as suggested by (AL-Jawary, 2017; Siddiqui et al., 2012a). The approximations by the DJM for this case are

$$w_0 = 1 + 0.3\left(-1+\frac{x}{2}\right)x,$$

$$w_1 = -1.(-0.027x + 0.0405x^2 - 0.027x^3 + 0.00675x^4),$$

$$\begin{aligned} w_2 = 1.(-0.027x + 0.0405x^2 - 0.027x^3 + 0.00675x^4) \\ -1.(-0.020346416999999995x + 0.024482776499999997x^2 \\ -0.007056503999999997x^3 - 0.006147468000000006x^4 \\ +0.003193311599999997x^5 + 0.0006680070000000001x^6 \\ -0.0004199039999999999x^7 - 0.0000065610000000000002x^8 \\ +0.00001968299999999998x^9 - 0.0000019683x^{10}), \end{aligned}$$

$$\vdots$$

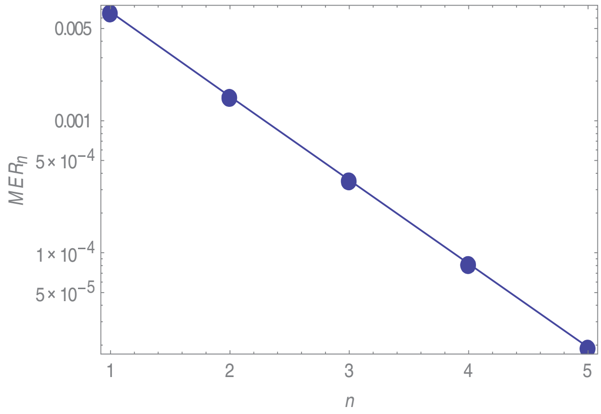


Figure 1. The logarithmic plots of MER_n by DJM when $\beta = 0.5$ and $m = 0.3$.

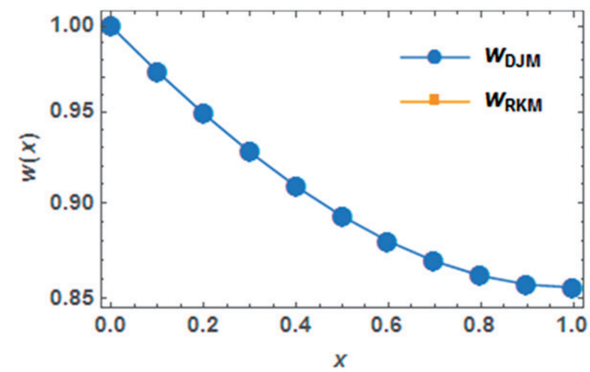


Figure 2. Comparison between the curves of the approximate series function by DJM and the numerical function which obtained by RKM for $0 \le x \le 1$ when $\beta = 0.5$ and $m = 0.3$.

The logarithmic plots of the maximum error remainder parameters MER_n, for $n = 1$ through 5 are shown in Figure 1 where an exponential rate of convergence can be seen. To show the validity of the DJM; Figure 2 shows the difference between the approximate solution, which is produced by the DJM and the numerical solution that is evaluated by using the RKM (AL-Jawary, 2017).

To show the validity for the DJM in reaching the best accuracy for the obtained approximate solutions, we have used the root mean square (RMS) norm to evaluate the difference between the solutions of the DJM and RKM. For this matter, the RMS

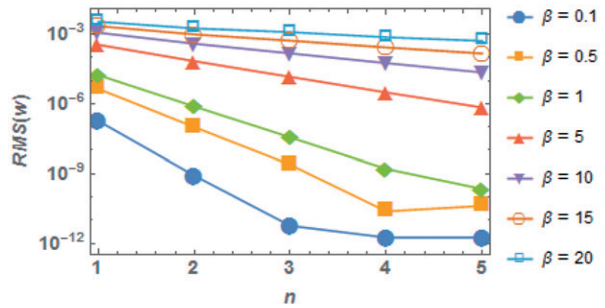


Figure 3. The curves of the RMS differences at different values of β when $m = 0.1$.

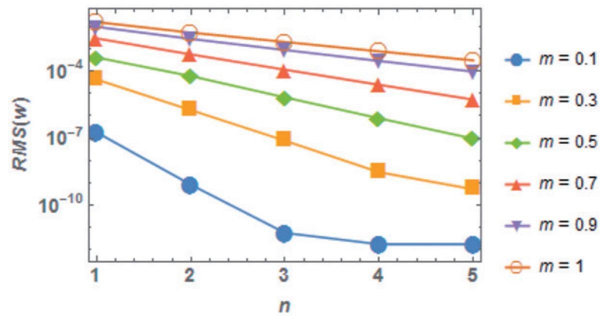


Figure 4. The curves of the RMS differences at different values of m when $\beta = 0.1$.

is given in the following form

$$RMS(w) = \sqrt{\frac{\sum (w_{DJM} - w_{RKM})^2}{\sum (w_{RKM})^2}}, \tag{35}$$

Figures 3 and 4 show the RMS differences versus n. We observe good convergence in the RMS curves as the value of n increases. At constant m note that the higher the β value, the larger the RMS difference, as shown in Figure 3. Also, keeping $\beta = 0.1$ with increasing the values of m will make the convergence poorer (Figure 4). In all cases we may conclude that the approximate DJM solution becomes more accurate whenever n increases. The rate of convergence with increasing n for the case of $\beta = 0.5$ and $m = 0.3$ was estimated using $\log(MER_4/MER_3)/\log(MER_3/MER_2) = 1.0$ proving linear convergence of the method.

6. Numerical comparisons

In this section, we present a comparison between our approximate solution obtained using the DJM and the approximate solutions obtained by previous studies using the ADM, VIM-I, VIM-II and VIM-III methods. In comparison, the ADM requires to evaluation the Adomian polynomials, which are computationally expensive. When comparing the DJM with the VIM-j; we find that there is no need for evaluating the Lagrange multipliers in DJM, which requires additional calculations when using the VIM-I, VIM-II

Table 1. The MER_5 for the solutions of the ADM, VIM-I and DJM for different values of β when $m = 0.3$.

β	ADM	VIM-*I*	DJM
0.1	1.39694×10^{-8}	4.06296×10^{-8}	2.11964×10^{-9}
0.2	8.59592×10^{-7}	1.15823×10^{-6}	1.17068×10^{-7}
0.3	9.43745×10^{-6}	7.88187×10^{-6}	1.16024×10^{-6}
0.4	0.0000512	0.0000299	5.71314×10^{-6}
0.5	0.000189	0.0000826	0.000019
1	0.010431	0.001694	0.000707

Table 2. The MER_5 for the solutions of the ADM, VIM-I and DJM for different values of m when $\beta = 0.5$.

m	ADM	VIM-I	DJM
0.1	1.39442×10^{-10}	7.56112×10^{-10}	2.22449×10^{-11}
0.2	1.06931×10^{-6}	1.27554×10^{-6}	1.42288×10^{-7}
0.3	0.000189099	0.0000826431	0.0000192222
0.4	0.00708853	0.00137525	0.000521409
0.5	0.108326	0.0107724	0.00584935

and VIM-III methods. Furthermore, the final solution in the DJM is based on the sum of resulting iterative terms. In contrast, the VIM-I approximate solution is obtained by taking the limit of the resulting successive approximations. Tables 1 and 2 present the error norm MER_5 for the solutions obtained by the ADM, VIM-I and DJM. It can be clearly seen that the best accuracy is obtained by the DJM numerical solution. The values of the MER_n for the fifth order approximate solutions is express smaller error of DJM in comparison to ADM and VIM-I.

Finally, when comparing the DJM with the other numerical methods, especially Runge–Kutta method (RKM); there is no need to use any type of truncation errors to measure the accuracy of the obtained approximate solution. There is no need for resorting to any discretization processes or determining the step size of the subintervals over the whole interval in the DJM. Furthermore, there is no need for making any round-off errors. The only limitation comes from the physical properties of the underlying problem. As values of the parameters β and m are increased the nature of the problem changes and thus the error obtained at a specific n increases. Changing of the parameters has an effect on convergence rate as well.

7. Conclusions

In this work, we have derived an approximate solution of the thin film flow of a non-Newtonian fluid by applying the Daftardar-Jafari iterative method. The DJM does not require any restricted assumptions, as they are required when using other iterative methods such as VIM or HAM. Furthermore, there is no need to resort to additional calculations such as evaluating Adomian polynomials as in the case of ADM. The differences and similarities between DJM and VIM were explored in detail, highlighting the most important ones. By examining convergence properties of DJM for several parameter values of the thin film fluid flow problem, we observe good convergence properties. However, we did find that the choice of the parameters does have an effect on convergence.

Acknowledgements

The author would like to thank the anonymous referees, Managing Editor and Editor in Chief for their valuable suggestions.

Disclosure statement

The authors declare that there is no conflict of interest.

References

Adomian, G. (1994). *Solving frontier problems of physics: The decomposition method*. Boston, MA: Kluwer Academic Publishers.

AL-Jawary, M. A. (2016). An efficient iterative method for solving the Fokker–Planck equation. *Results in Physics*, *6*, 985–991. https://doi.org/10.1016/j.rinp.2016.11.018

AL-Jawary, M. A. (2017). A semi-analytical iterative method for solving nonlinear thin film flow problems. *Chaos, Solitons and Fractals*, *99*, 52–56. https://doi.org/10.1016/j.chaos.2017.03.045

AL-Jawary, M. A., & AL-Qaissy, H. R. (2015). A reliable iterative method for solving Volterra integro-differential equations and some applications for the Lane–Emden equations of the first kind. *Monthly Notices of the Royal Astronomical Society*, *448*, 3093–3104. doi:10.1093/mnras/stv198

Al-Jawary, M. A., & Al-Razaq, S. G. (2016). A semi analytical iterative technique for solving duffing equations. *International Journal of Pure and Applied Mathematics*, *108*, 871–885. doi:10.12732/ijpam.v108i4.13

Al-Jawary, M. A., & Raham, R. K. (2016). A semi-analytical iterative technique for solving chemistry problems. *Journal of King Saud University Science*, *29*, 320–332. https://doi.org/10.1016/j.jksus.2016.08.002

Banach, S. (1922). Sur les opérations dans les ensembles abstraits et leur application aux equations integrals. *Fundamenta Mathematicae*, *3*, 133–181. https://eudml.org/doc/213289

Bhalekar, S., & Daftardar-Gejji, V. (2008). New iterative method: Application to partial differential equations. *Applied Mathematics and Computation*, *203*, 778–783. https://doi.org/10.1016/j.amc.2008.05.071

Bhalekar, S., & Daftardar-Gejji, V. (2011). Convergence of the new iterative method. *International Journal of Differential Equations*, 2011. doi:10.1155/2011/989065

Bhalekar, S., & Daftardar-Gejji, V. (2012). Solving a system of nonlinear functional equations using revised new iterative method. *Proceedings of World Academy of Science, Engineering and Technology*, *68*, 08–21. doi:10.1999/1307-6892/1209

Bhatti, M. M., Zeeshan, A., & Ellahi, R. (2016). Endoscope analysis on peristaltic blood flow of Sisko fluid with Titanium magneto-nanoparticles. *Computers in biology and medicine*, *78*, 29–41. doi:10.1016/j.compbiomed.2016.09.007

Biazar, J., & Ghazvini, H. (2009). Convergence of the homotopy perturbation method for partial differential equations. *Nonlinear Analysis Real World Applications*, *10*, 2633–2640. https://doi.org/10.1016/j.nonrwa.2008.07.002

Bildik, N., Konuralp, A., Bek, F., & Kucukarslan, S. (2006). Solution of different type of the partial differential equation by differential transform method and Adomian's decomposition method. *Applied Mathematics and Computation*, *172*, 551–567. https://doi.org/10.1016/j.amc.2005.02.037

Daftardar-Gejji, V., & Bhalekar, S. (2010). Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method. *Computers and Mathematics with Applications*, *59*, 1801–1809. https://doi.org/10.1016/j.camwa.2009.08.018

Daftardar-Gejji, V., & Jafari, H. (2006). An iterative method for solving nonlinear functional equations. *Journal of Mathematical Analysis and Applications*, *316*, 753–763. https://doi.org/10.1016/j.jmaa.2005.05.009

Gul, T., Islam, S., Shah, R. A., Khan, I., & Shafie, S. (2014). Thin film flow in MHD third grade fluid on a vertical belt with temperature dependent viscosity. *PLoS One*, *9*, e97552. doi:10.1371/journal.pone.0097552.

He, J. H. (1999a). Homotopy perturbation technique. *Computer Methods in Applied Mechanics and Engineering*, *178*, 257–262. https://doi.org/10.1016/S0045-7825(99)00018-3

He, J. H. (1999b). Variational iteration method-a kind of non-linear analytical technique: some examples. *International Journal of Non Linear Mechanics*, *34*, 699–708. https://doi.org/10.1016/S0020-7462(98)00048-1

He, J. H. (2000). A coupling method of a homotopy technique and a perturbation technique for non-linear problems. *International Journal of Non-Linear Mechanics*, *35*, 37–43. https://doi.org/10.1016/S0020-7462(98)00085-7

He, J. H., Wu, G. C., & Austin, F. (2010). The variational iteration method which should be followed. *Nonlinear Science Letters A*, *1*, 1–30. https://works.bepress.com/ji_huan_he/49/

He, J. H. (2012). Notes on the optimal variational iteration method. *Applied Mathematics Letters*, *25*, 1579–1581. https://doi.org/10.1016/j.aml.2012.01.004

Hemeda, A. A. (2013). New iterative method: An application for solving fractional physical differential equations. Abstract and Applied Analysis, 2013. http://dx.doi.org/10.1155/2013/617010

Liao, S. J. (2004). *Beyond perturbation: Introduction to homotopy analysis method*. Boca Raton: Chapman & Hall/CRC Press.

Mabood, F. (2014). Comparison of optimal homotopy asymptotic method and homotopy perturbation method for strongly non-linear equation. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *16*, 21–26. https://doi.org/10.1016/j.jaubas.2013.07.002

Mabood, F., & Pochai, N. (2015). Comparison of optimal homotopy asymptotic and Adomian decomposition methods for a thin film flow of a third grade fluid on a moving belt. *Advances in Mathematical Physics*, 2015. doi:10.1155/2015/642835.

Moosavi, M., Momeni, M., Tavangar, T., Mohammadyari, R., & Rahimi-Esbo, M. (2016). Variational iteration method for flow of non-Newtonian fluid on a moving belt and in a collector. *Alexandria Engineering Journal*, *55*, 1775–1783. https://doi.org/10.1016/j.aej.2016.03.033

Nemati, H., Ghanbarpour, M., Hajibabayi, M., & Hemmatnezhad. (2009). Thin film flow of non-Newtonian fluids on a vertical moving belt using homotopy analysis method. *Journal of Engineering Science and Technology Review*, *2*, 118–122. https://doaj.org/article/63c97a2d6cb84b3086118e3b1b70b228

Rajagopal, K. R. (1983). A note on unsteady unidirectional flows of a non-Newtonian fluid. *International Journal of Non-Linear Mechanics*, *17*, 369–373. https://doi.org/10.1016/0020-7462(82)90006-3

Rashidi, M. M., Bagheri, S., Momoniat, E., & Freidoonimehr, N. (2017). Entropy analysis of convective MHD flow of third grade non-Newtonian fluid over a stretching sheet. *Ain Shams Engineering Journal*, *8*, 77–85. https://doi.org/10.1016/j.asej.2015.08.012

Ravnik, J., & Skerget, L. (2015). A numerical study of nanofluid natural convection in a cubic enclosure with a circular and an ellipsoidal cylinder. *International journal of heat and mass transfer*, *89*, 596–605. https://doi.org/10.1016/j.ijheatmasstransfer.2015.05.089

Sajid, M., & Hayat, T. (2008). The application of homotopy analysis method to thin film flows of a third order fluid. *Chaos, Solitons and Fractals*, *38*, 506–515. doi:10.1016/j.chaos.2006.11.034

Shah, H., Pandya, J., & Shah, P. (2016). Approximate solution for the thin film flow problem of a third grade fluid using spline collocation method. *International Journal of Advances in Applied Mathematics and Mechanics*, *1*, 91–97. https://dx.doi.org/10.22606/jaam.2016.12001

Sheikholeslami, M., & Zeeshan, A. (2017). Mesoscopic simulation of CuO–H_2O nanofluid in a porous enclosure with elliptic heat source. *International Journal of Hydrogen Energy*, *42*, 15393–15402. doi:10.1016/j.ijhydene.2017.04.276

Siddiqui, A. M., Farooq, A. A., Haroon, T., & Babcock, B. S. (2012a). A comparison of variational iteration and Adomian decomposition methods in solving nonlinear thin film flow problems. *Applied Mathematical Sciences*, *6*, 4911–4919. http://www.m-hikari.com/ams/ams-2012/ams-97-100-2012/babcockAMS97-100-2012-2.pdf

Siddiqui, A. M., Farooq, A. A., Haroon, T., Rana, M. A., & Babcock, B. S. (2012b). Application of He's variational iterative method for solving thin film flow problem arising in non-Newtonian fluid mechanics. *World Journal of Mechanics*, *2*, 138–142. doi:10.4236/wjm.2012.23016

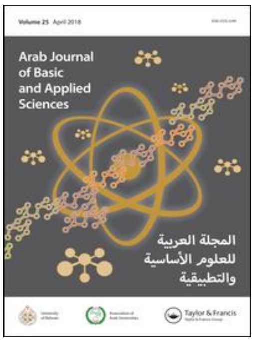
Siddiqui, M., Hameed, M. A., Siddiqui, B. M., & Ghori, Q. K. (2010). Use of Adomian decomposition method in the study of parallel plate flow of a third grade fluid. *Communications in Nonlinear Science and Numerical Simulation*, *15*, 2388–2399. https://doi.org/10.1016/j.cnsns.2009.05.073

Yaseen, M., Samraiz, M., & Naheed, S. (2013). Exact solutions of Laplace equation by DJ method. *Results in physics*, *3*, 38–40. https://doi.org/10.1016/j.rinp.2013.01.001

Zeeshan, A., & Atlas, M. (2017). Optimal solution of integro-differential equation of Suspension Bridge Model using Genetic Algorithm and Nelder-Mead method. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *24*, 310–314. https://doi.org/10.1016/j.jaubas.2017.05.003

Zeeshan, A., Majeed, A., & Ellahi, R. (2016). Effect of magnetic dipole on radiative non-darcian mixed convective flow over a stretching sheet in porous medium. *Journal of Nanofluids*, *5*, 617–626. doi:10.1166/jon.2016.1237

Zhou, J. K. (1986). *Differential Transformation and its Applications for Electrical Circuits*. Wuhan, China: Huazhong Univ. Press. (in Chinese).



Taylor & Francis
Taylor & Francis Group

Arab Journal of Basic and Applied Sciences

ISSN: (Print) 2576-5299 (Online) Journal homepage: http://www.tandfonline.com/loi/tabs20

Three-dimensional analysis of condensation nanofluid film on an inclined rotating disk by efficient analytical methods

M. Hatami, D. Jing & Majeed A. Yousif

To cite this article: M. Hatami, D. Jing & Majeed A. Yousif (2018) Three-dimensional analysis of condensation nanofluid film on an inclined rotating disk by efficient analytical methods, Arab Journal of Basic and Applied Sciences, 25:1, 28-37, DOI: 10.1080/25765299.2018.1449415

To link to this article: https://doi.org/10.1080/25765299.2018.1449415

Accepted author version posted online: 09 Apr 2018.
Published online: 23 Apr 2018.

ARAB JOURNAL OF BASIC AND APPLIED SCIENCES
2018, VOL. 25, NO. 1, 28–37
https://doi.org/10.1080/25765299.2018.1449415

University of Bahrain
Taylor & Francis
Taylor & Francis Group

ORIGINAL ARTICLE

OPEN ACCESS Check for updates

Three-dimensional analysis of condensation nanofluid film on an inclined rotating disk by efficient analytical methods

M. Hatami[a,b], D. Jing[b] and Majeed A. Yousif[c]

[a]Department of Mechanical Engineering, Esfarayen University of Technology, Esfarayen, Iran; [b]State Key Laboratory of Multiphase Flow in Power Engineering & International Research Center for Renewable Energy, Xi'an Jiaotong University, Xi'an, China; [c]Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Iraq

ABSTRACT
In this study, least square method (LSM) and differential transform method (DTM) applied to solve the three dimensional problem of steady nanofluid deposition on an inclined rotating disk is illustrated. The governing non-linear partial differential equations are reduced to the nonlinear ordinary differential equations system by similarity transform. There is a good agreement between the present analytical and numerical results. Results indicate that increasing nanofluid volume fraction leads to increase in temperature profile and highest temperature obtained for aluminium oxide (Al_2O_3)-water case.

ARTICLE HISTORY
Received 13 March 2017
Accepted 27 August 2017

KEYWORDS
Nanofluid; rotating disk; condensation film; film thickness; LSM; DTM

1. Introduction

The removal of a liquid condensate from a cooled, saturated vapor is important in chemical and mechanical engineering processes. Many researchers illustrated this problem at the different conditions. Sparrow and Gregg (1959) studied the removal of the condensate using centrifugal forces on a cooled rotating disc. Sparrow and Gregg (1959) transformed the Navier-Stokes equations into a set of nonlinear ordinary differential equations and numerically integrated for the similarity solution for several finite film thicknesses. The problem is also related to chemical vapor deposition, when a thin fluid film is deposited on a cooled rotating disk (Jensen, Einset, & Fotiadis, 1991).

Enhancement of heat transfer performance in these systems is an essential topic from an energy saving perspective. The low thermal conductivity of conventional heat transfer fluids such as water and oils is a primary limitation in enhancing the performance and the compactness of such systems. Solid typically has a higher thermal conductivity than liquids. For example, copper (Cu) has a thermal conductivity 700 time greater than water and 3000 greater than engine oil. An innovative and new technique to enhance heat transfer is by using solid particles in the base fluid (i.e. nanofluids) in the range of sizes 10–50 nm. Khanafer, Vafai, and Lightstone (2003) firstly conducted a numerical investigation on the heat transfer enhancement by adding nano-particles in a differentially heated enclosure. They found that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. Rashidi, Abelman, and Mehr (2013) considered the analysis of the second law of thermodynamics applied to an electrically conducting incompressible nanofluid fluid flowing over a porous rotating disk. They concluded that using magnetic rotating disk drives has important applications in heat transfer enhancement in renewable energy systems. Ellahi (2013) studied the magnetohydrodynamic (MHD) flow of non-Newtonian nanofluid in a pipe. He observed that the MHD parameter decreases the fluid motion and the velocity profile is larger than that of temperature profile even in the presence of variable viscosities. Free convection heat transfer in a concentric annulus between a cold square and heated elliptic cylinders in presence of magnetic field was investigated by Sheikholeslami, Gorji-Bandpy, and Ganji (2013a). They found that the enhancement in heat transfer increases as Hartmann number increases but it decreases with increase of Rayleigh number. Asymmetric laminar flow and heat transfer of nanofluid between contracting rotating disks was investigated by Hatami, Sheikholeslami, and Ganji (2014). Their results indicated that temperature profile becomes more flat near the middle of two disks with the increase of

CONTACT M. Hatami m.hatami2010@gmail.com State Key Laboratory of Multiphase Flow in Power Engineering & International Research Center for Renewable Energy, Xi'an Jiaotong University, Xi'an 710049, China

injection but an opposite trend is observed with increase of expansion ratio. The problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field was investigated analytically by Sheikholeslami, Hatami, and Ganji, (2013b). Their results showed that velocity boundary layer thickness decrease with increase of Reynolds number and it increase as Hartmann number increases. Khan and Pop (2010) published a paper on boundary-layer flow of a nanofluid past a stretching sheet. They indicated that the reduced Nusselt number is a decreasing function of each dimensionless number. Hassani, Tabar, Nemati, Domairry, and Noori (2011) investigated the problem of boundary layer flow of a nanofluid past a stretching sheet. They found that the reduced Nusselt number decreases with the increase in Prandtl number for many Brownian motion numbers. Rashidi and Dinarvand (2009) investigated the steady three-dimensional problem of condensation film on inclined rotating disk by analytical methods introduced by Aziz (2006) using Maple software where Bhatti and Zeeshan (2017), Bhatti, Zeeshan, and Ellahi (2016, 2017a), Bhatti, Zeeshan, Ellahi, and Ijaz (2017b), Ezzat (1994, 2001, 2011, 2012), Ezzat and Youssef (2010), Hatami and Ganji (2014a, 2014b), MA. Ezzat, El-Bary, and SM. Ezzat (2013); MA. Ezzat, Abbas, El-Bary, and SM. Ezzat (2014a), M. Ezzat, Sabbah, El-Bary, and S. Ezzat (2014b), Nawaz, Hayat, and Zeeshan (2016), Othman and Ezzat (2001), Zeeshan, Hassan, Ellahi, and Nawaz (2016), Zeeshan and Majeed (2016a, 2016b), and some recent researchers (Devakar, Ramesh, Chouhan, & Raje, 2017; Mahanthesh, Gireesha, & Gorla, 2017; Mirgolbabaee, Ledari, & Ganji, 2017; Saleh, Alali, & Ebaid, 2017; Srinivasacharya & Shafeeurrahman, 2017) used these valuable methods for analyzing the heat transfer and nanofluid flow problems in different geometries.

In this work we shall develop and apply least square method (LSM) and differential transform method (DTM) to solve the problem of three-dimensional analysis of condensation nanofluid film on an inclined rotating disk.

2. Flow analysis and mathematical formulation

Figure 1 (Rashidi & Dinarvand, 2009) shows a disk rotating in its own plane with angular velocity Ω. The angle between horizontal axis and disk is β. A nanofluid film of thickness h is formed by spraying, with the W velocity. We assume the disk radius is large compared to the film thickness such that the end effects can be ignored. Vapor shear effects at the interface of vapor and fluid are usually

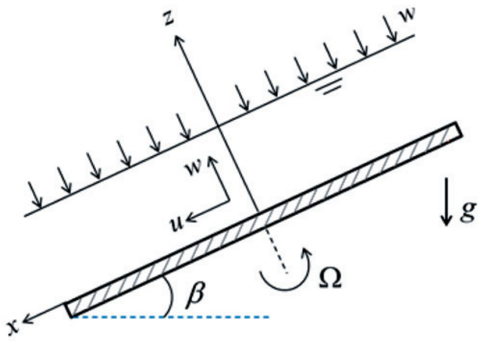


Figure 1. Schematic diagram of the problem.

Table1. Thermophysical properties of water and the nanoparticles.

	Pure water	Cu	CuO	Al_2O_3
$C_p\,(j/kgk)$	4179	383	540	765
$\rho\,(kg/m^3)$	997.1	8954	6500	3970
$k\,(W/m.k)$	0.613	400	18	25

unimportant. The gravitational acceleration, $\bar{g}$, acts in the downward direction.

The temperature on the disk is T_w and the temperature on the film surface is T_0. Besides, the ambient pressure on the film surface is constant at p_0 and we can safely say the pressure is a function of z only.

The nanofluid is a two component mixture with the following assumptions: incompressible; no-chemical reaction; negligible radiative heat transfer; nano-solid-particles and the base fluid are in thermal equilibrium with no slip between them. The thermo physical properties of the nanofluid are given in Table 1.

Neglecting viscous dissipation, the continuity, momentum and energy equations for steady state are given in the Equations (1–5):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\bar{g}\sin\beta \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\bar{g}\cos\beta-\frac{p_z}{\rho_{nf}} \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

In the above equations, u, v, and w indicate the velocity components in the x, y, and z directions, respectively.

The effective density (ρ_{nf}), the effective heat capacity $(\rho C_p)_{nf}$ of the nanofluid and the effective heat capacity $(\rho C_p)_{nf}$ of the nanofluid are defined by Equation (6) (Khanafer et al., 2003):

$$\begin{aligned} &\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \\ &(\rho\, C_p)_{nf} = (1-\phi)\ (\rho\, C_p)_f + \phi(\rho\, C_p)_s \end{aligned} \tag{6}$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell-Garnett's (MG) model as given in Equation (7) (Jensen et al., 1991):

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \tag{7}$$

Supposing zero slip on the disk and zero shear stress on the film surface, the boundary conditions are represented as Equation (8):

$$u = -\Omega y,\ v = \Omega x,\ w = 0,\ T = T_w \quad at \quad z = 0 \tag{8}$$

If

$$u_z = 0,\ v_z = 0,\ w = -W,\ T = T_0,\ p = p_0,\ at \quad z = h$$

Wang introduced the following transform as given in Equation (9) (Jensen et al., 1991):

$$\begin{aligned} &u = -\Omega y\ g(\eta) + \Omega x\ f'(\eta) + \bar{g}\ k(\eta)\ \sin\ \frac{\beta}{\Omega'} \\ &v = \Omega x\ g(\eta) + \Omega y\ f'(\eta) + \ \bar{g}\ s(\eta)\sin\ \frac{\beta}{\Omega'} \\ &w = -2\sqrt{\Omega \upsilon_{nf}}\ f(\eta) \\ &T = (T_0 - T_w)\ \theta(\eta) + T_w \end{aligned} \tag{9}$$

where η was introduced as given in Equation (10):

$$\eta = z\sqrt{\frac{\Omega}{\upsilon_{nf}}} \tag{10}$$

When Equation (1) automatically is satisfied, then Equations (2) and (3) can be rewritten as Equations (11–14):

$$f''' - (f')^2 + g^2 + 2ff'' = 0 \tag{11}$$

$$g'' - 2gf' + 2fg' = 0 \tag{12}$$

$$k'' - kf' + sg + 2fk' + 1 = 0 \tag{13}$$

$$s'' - kg - sf' + 2fs' = 0 \tag{14}$$

If the temperature is a function of the distance z only, then Equation (5) can be rewritten as given in Equation (15)

$$\begin{aligned} &\theta'' + 2\,\mathrm{Pr}\frac{A_2\ A_3}{A_1\ A_4} f\,\theta' = 0, \\ &A_1 = \frac{\rho_{nf}}{\rho_f},\ A_2 = \frac{\mu_{nf}}{\mu_f},\ A_3 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f},\ A_4 = \frac{k_{nf}}{k_f}. \end{aligned} \tag{15}$$

where $\mathrm{Pr} = \frac{\upsilon_f}{\alpha_f}$ is the Prandtl number of base fluid. The boundary conditions for Equations (11–15) areas given in Equation (16):

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 0, \quad f''(\delta) = 0, \\ &g(0) = 1, \quad g'(\delta) = 0, \\ &k(0) = 0, \quad k'(\delta) = 0, \\ &s(0) = 0, \quad s'(\delta) = 0, \\ &\theta(0) = 0, \quad \theta(\delta) = 1 \end{aligned} \tag{16}$$

and δ is the constant normalized thickness as presented in Equation (17):

$$\delta = h\sqrt{\frac{\Omega}{\upsilon_{nf}}} \tag{17}$$

which is known through the condensation or spraying velocity as given in Equation (18):

$$f(\delta) = \frac{W}{2\sqrt{\Omega\ \upsilon_{nf}}} = \alpha \tag{18}$$

In this case the non-dimensional Nusselt number is obtained using Equation (19):

$$Nu = \frac{k_{nf}}{k_f} \frac{\left(\frac{\partial T}{\partial z}\right)_w}{T_0 - T_w} = A_4\delta\ \theta'(0) \tag{19}$$

3. Numerical and analytical applied methods

3.1. Numerical approaches

3.1.1 Fourth-order Runge-Kutta-Fehlberg method

As already mentioned, the current problem type is boundary value problem (BVP) and an appropriate method needs to be selected for this. The available sub-methods in the Maple 15.0 are a combination of the base schemes; trapezoid or midpoint method. There are two major considerations when choosing a method for a problem. The trapezoid method is generally efficient for typical problems, but the midpoint method is capable of handling harmless end-point singularities whereas the trapezoid method cannot. The midpoint method, also known as the fourth-order Runge-Kutta-Fehlberg method, improves the Euler method by adding a midpoint in the step which increases the accuracy by one order. Thus, the midpoint method is used as a suitable numerical technique in this approach. (Aziz, 2006).

3.1.2 Shooting method

Here mathematic10.0 software have been applied to solve BVP by using shooting numerical method. The method will shoot out paths in various directions until a path that has the required boundary value is found. Shooting method is used to compare the results with results obtained via DTM.

3.2 Analytical solution

3.2.1 Least square method

LSM is one of the approximation techniques for solving differential equations and this method is also called the weighted residual methods (WRMs). Here a differential operator D is acted on a function u to produce a function p (Hatami & Ganji, 2014a, 2014b) as shown in Equation (20):

$$D(u(x)) = p(x) \tag{20}$$

It is considered that u is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is represented in Equation (21) as,

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{21}$$

Now, when substituted into the differential operator, D, the result of the operations generally isn't $p(x)$. Hence an error or residual will exist as shown in Equation (22):

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{22}$$

The notion in WRMs is to force the residual to zero in some average sense over the domain. That is written as Equation (23):

$$\int_X R(x)\ W_i(x) = 0 \qquad i = 1, 2, \ldots, n \tag{23}$$

where the number of weight functions W_i is exactly equal to the number of unknown constants c_i in $\tilde{u}$. If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of Equation 24 can be seen.

$$S = \int_X R(x)R(x)dx = \int_X R^2(x)dx \tag{24}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is shown in Equation (25),

$$\frac{\partial S}{\partial c_i} = 2\int_X R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{25}$$

Comparing with Equation (23), the weight functions are seen to be as represented in Equation (26)

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{26}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the LSM are just the derivatives of the residual with respect to the unknown constants as given in Equation (27):

$$W_i = \frac{\partial R}{\partial c_i} \tag{27}$$

Now, we want to apply this method to the present problem. Because trial functions must satisfy the boundary conditions in Equation (16), so they will be considered as Equation (28),

$$\begin{cases} f(\eta) = c_1\eta^2(\eta-\delta)^3 + c_2\left(\dfrac{\eta^3}{6} - \dfrac{\delta\eta^2}{2}\right) + \ldots \\ g(\eta) = 1 + c_3\eta(\eta-\delta)^2 + c_4\left(\dfrac{\eta^2}{2} - \delta\eta\right) + \ldots \\ k(\eta) = c_5\eta(\eta-\delta)^2 + c_6\left(\dfrac{\eta^2}{2} - \delta\eta\right) + \ldots \\ s(\eta) = c_7\eta(\eta-\delta)^2 + c_8\left(\dfrac{\eta^2}{2} - \delta\eta\right) + \ldots \\ \theta(\eta) = \dfrac{\eta}{\delta} + c_9\eta(\eta-\delta) + c_{10}\eta(\eta-\delta)^2 + \ldots \end{cases} \tag{28}$$

It's necessary to inform that every trial function that satisfy the boundary condition of the problem can be used and its accuracy can be improved by the number of its terms. In this problem, we have five coupled equations (Equations (11–15)) so, five residual functions will appear. Also, in this article we considered two unknown coefficient for each trial function, so 10 unknown coefficients will be seen (c_1–c_{10}). By substituting the residual functions, $R_1(c_1–c_{10}, \eta)$, $R_2(c_1–c_{10}, \eta)$, $R_3(c_1–c_{10}, \eta)$, $R_4(c_1–c_{10}, \eta)$ *and* $R_5(c_1–c_{10}, \eta)$, into Equation (25), a set of ten equations will appear and by solving this system of equations, coefficients c_1–c_{10} will be determined. For example, using LSM for Cu-water nanofluid with $\phi = 0.04$, $Pr = 6.2$ and $\delta = 0.5$ Equation (29) will be obtained

$$\begin{cases} f(\eta) = -0.008811728449\eta^2(\eta-0.5)^3 - 0.9607426021\left(\dfrac{\eta^3}{6} - \dfrac{\eta^2}{4}\right) \\ g(\eta) = 1 + 0.07794439347\eta(\eta-0.5)^2 + 0.1969431465\left(\dfrac{\eta^2}{2} - \dfrac{\eta}{2}\right) \\ k(\eta) = 0.008808870176\eta(\eta-0.5)^2 - 0.9838801251\left(\dfrac{\eta^2}{2} - \dfrac{\eta}{2}\right) \\ s(\eta) = 0.03931075727\eta(\eta-0.5)^2 + 0.09985540140\left(\dfrac{\eta^2}{2} - \dfrac{\eta}{2}\right) \\ \theta(\eta) = 2\eta - 0.1982408923\eta(\eta-0.5) - 0.2492463454\eta(\eta-0.5)^2 \end{cases} \tag{29}$$

In the same manner for Cu-water nanofluid when $\delta = 1$ it can represented as Equation (30):

$$\begin{cases} f(\eta) = -0.01914194896\eta^2(\eta-1)^3 - 0.6564549829\left(\dfrac{\eta^3}{6} - \dfrac{\eta^2}{2}\right) \\ g(\eta) = 1 + 0.08162945698\eta(\eta-\delta)^2 + 0.4634992471\left(\dfrac{\eta^2}{2} - \eta\right) \\ k(\eta) = 0.03926545013\eta(\eta-1)^2 - 0.8513453189\left(\dfrac{\eta^2}{2} - \eta\right) \\ s(\eta) = 0.04388591688\eta(\eta-1)^2 + 0.2695506788\left(\dfrac{\eta^2}{2} - \eta\right) \\ \theta(\eta) = \eta - 0.4379485922\eta(\eta-1) - 0.2403810627\eta(\eta-1)^2 \end{cases} \tag{30}$$

Many advantages of LSM compared to other analytical and numerical methods make it more valuable and motivate researchers to use it for solving heat transfer problems. Some of these advantages are listed below (Hatami & Ganji, 2014a):

a. It solves the equations directly and no further simplifications needs to be done. For example it solves power nonlinear terms without expanding or using Taylor expansion against differential transformation method (DTM).
b. It does not require any perturbation, linearization or small parameter versus homotopy perturbation method (HPM) and parameter perturbation method (PPM).
c. It is simple and powerful compared to the other numerical methods and reaches the final results faster than numerical procedures. Furthermore while its results are acceptable and have excellent agreement with numerical outcomes, its accuracy can be increased by increasing the statements of the trial functions.
d. It does not require to determine the auxiliary parameter and auxiliary function versus HAM.

3.2.2 Differential transform method

Equations (11–15) is transformed to one-dimensional differential transform, using Equation (16) which has resulted in Equations (31–45) Zhou (1986):

$$f''' \to (i+3)(i+2)(i+1)F(i+3) \tag{31}$$

$$ff'' \to \sum_{r=0}^{i}(i+2-r)(i+1-r)F(r)F(i+2-r) \tag{32}$$

$$f'^2 \to \sum_{r=0}^{i}(i+1-r)(r+1)F(r+1)F(i+1-r) \tag{33}$$

$$g'' \to (i+2)(i+1)G(i+2) \tag{34}$$

$$k'' \to (i+2)(i+1)K(i+2) \tag{35}$$

$$s'' \to (i+2)(i+1)S(i+2) \tag{36}$$

$$\theta'' \to (i+2)(i+1)\Theta(i+2) \tag{37}$$

$$g^2 \to \sum_{r=0}^{i}G(r)G(i-r) \tag{38}$$

$$fg' \to \sum_{r=0}^{i}(i+1-r)F(r)G(i+1-r) \tag{39}$$

$$gf' \to \sum_{r=0}^{i}(i+1-r)G(r)F(i+1-r) \tag{40}$$

$$fk' \to \sum_{r=0}^{i}(i+1-r)F(r)K(i+1-r) \tag{41}$$

$$kf' \to \sum_{r=0}^{i}(i+1-r)K(r)F(i+1-r) \tag{42}$$

$$fs' \to \sum_{r=0}^{i}(i+1-r)F(r)S(i+1-r) \tag{43}$$

$$sf' \to \sum_{r=0}^{i}(i+1-r)S(r)F(i+1-r) \tag{44}$$

$$f\theta' \to \sum_{r=0}^{i}(i+1-r)F(r)\Phi(i+1-r) \tag{45}$$

Substituting Equations (31–45) into Equations (11–15) according to boundary conditions as presented in Equation (16), we arrive at Equations (46–51):

$$(i+3)(i+2)(i+1)F(i+3) = -\begin{pmatrix} -\sum_{r=0}^{i}(i+1-r)(r+1)*F(r+1)F(i+1-r) \\ +\sum_{r=0}^{i}G(r)G(i-r) \\ +2\sum_{r=0}^{i}(i+1-r)(i+2-r)F(i+2-r)F(r) \end{pmatrix} \tag{46}$$

$$(i+2)(i+1)G(i+2) = -\begin{pmatrix} -2\sum_{r=0}^{i}(i+1-r)G(r)F(i+1-r) \\ +2\sum_{r=0}^{i}(i+1-r)F(r)G(i+1-r) \end{pmatrix} \tag{47}$$

$$(i+2)(i+1)K(i+2) = -\begin{pmatrix} -2\sum_{r=0}^{i}(i+1-r)K(r+1)F(i+1-r) \\ +\sum_{r=0}^{i}S(r)G(i-r) \\ +2\sum_{r=0}^{i}(i+1-r)F(r)K(i+1-r)+\delta_D(i) \end{pmatrix} \tag{48}$$

$$(i+2)(i+1)S(i+2) = \begin{pmatrix} -\sum_{r=0}^{i}G(r)K(i-r) \\ -\sum_{r=0}^{i}(i+1-r)S(r)F(i+1-r) \\ +2\sum_{r=0}^{i}(i+1-r)F(r)S(i+1-r) \end{pmatrix} \tag{49}$$

$$(i+2)(i+1)\Phi(i+2) = -2\frac{A_2\ A_3}{A_1\ A_4}Pr\sum_{r=0}^{i}(i+1-r)F(r+1)\Phi(i+1-r) \tag{50}$$

$$\begin{aligned} &F(0)=0;\ F(1)=0;\ F(2)=a_1 \\ &G(0)=1;\ G(1)=a_2 \\ &K(0)=0;\ K(1)=a_3 \\ &S(0)=0;\ S(1)=a_4 \\ &\Phi(0)=0;\ \Phi(1)=a_5 \end{aligned} \tag{51}$$

where $F(i)$, $G(i)$, $K(i)$, $S(k)$ and $\Phi(k)$ are the transformation functions of $f(i)$, $g(i)$, $k(i)$, $s(i)$ and $\theta(k)$ respectively and are defined by Equations (52–56):

$$f(\eta) = \sum_{i=0}^{\infty} F(i)\eta^i \tag{52}$$

$$g(\eta) = \sum_{i=0}^{\infty} G(i)\eta^i \tag{53}$$

$$k(\eta) = \sum_{i=0}^{\infty} K(i)\eta^i \tag{54}$$

$$s(\eta) = \sum_{i=0}^{\infty} S(i)\eta^i \tag{55}$$

$$\theta(\eta) = \sum_{k=0}^{\infty} \Phi(k)\eta^k \tag{56}$$

After using iteration $i = 0, 1, \ldots,$ and using Equations (46–51) and Equations (52–56), when $Pr = 1$, $\varphi = 0.4$ and $\delta = 1$, we obtained Equations (57–61):

$$f(\eta) = a_1\eta^2 - \frac{\eta^3}{6} - \frac{\eta^4 a_2}{12} - \frac{1}{60}\eta^5 a_2^2 + \ldots \tag{57}$$

$$g(\eta) = 1 + \frac{2\eta^3 a_1}{3} + \eta a_2 - \frac{\eta^5 a_2}{15} + \frac{1}{12}\eta^4\left(-2a_1a_2 + 2\left(-\frac{1}{2} + 2a_1a_2\right)\right) + \ldots \tag{58}$$

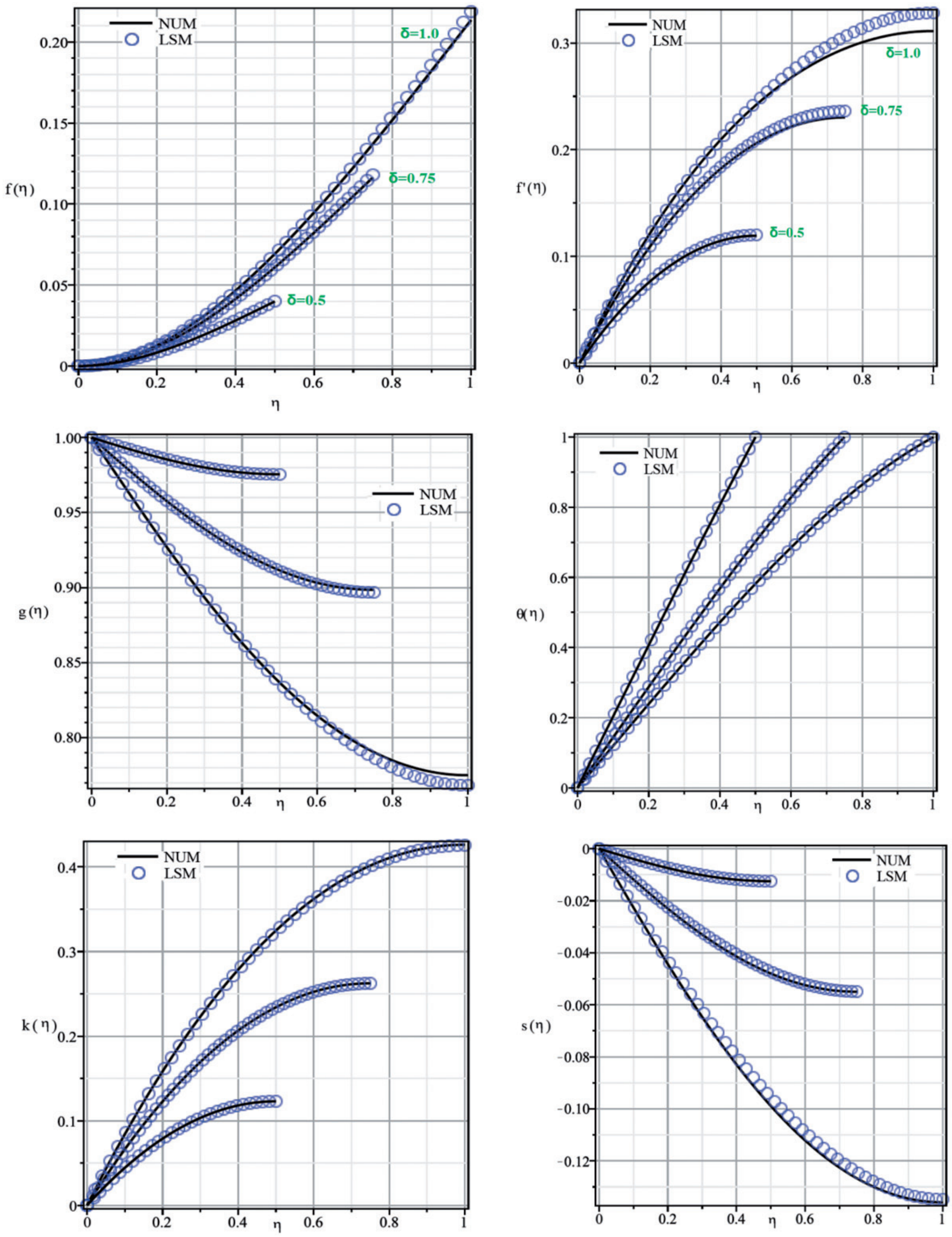


Figure 2. Comparison of LSM and NUM for Cu-water nanofluid with $\phi = 0.04$ and $\delta = 0.5$, 0.75 and 1, respectively.

$$k(\eta) = a_3\eta - \frac{\eta^2}{2} + \frac{1}{20}\eta^5\left(-1 - 2\left(-a_1 - \frac{a_3}{6}\right) - \frac{a_3}{6} + 2\left(\frac{1}{4} - \frac{a_2a_3}{3} + \frac{1}{3}a_1(-1 + 4a_1a_3 - a_4)\right)\right) + \frac{1}{6}\eta^3(-1 + 4a_1a_3 - a_4) + \ldots \tag{59}$$

$$s(\eta) = \frac{\eta^3 a_3}{6} + \frac{1}{12}\eta^4\left(-\frac{1}{2} + a_2a_3\right) + \frac{1}{20}\eta^5\left(-\frac{a_2}{2} + \frac{1}{6}(-1 + 4a_1a_3 - a_4) - \frac{a_4}{6}\right) + \eta a_4 + \ldots \tag{60}$$

$$\theta(\eta) = a_5\eta + \frac{\eta^4 a_5}{36} - \frac{1}{3}\eta^3 a_1 a_5 + \frac{1}{10}\eta^5\left(a_1^2 a_5 + \frac{a_2 a_5}{12}\right) + \ldots \tag{61}$$

On which we applied boundary conditions as given in Equation (16) to find a_i, $i = 1, 2, \ldots, 5$.

4. Results and discussions

The objective of the present study was to apply LSM to obtain an explicit analytic solution of three-dimensional problem of condensation nanofluid film on inclined rotating disk (Figure 1).

The comparison between the obtained results by LSM and DTM with that of numerical results by Maple and Mathematica are shown in Figures 2 and 3. This accuracy gives high confidence validity of this problem and reveals an excellent agreement of engineering accuracy for us. This investigation was completed by depicting the effects of some

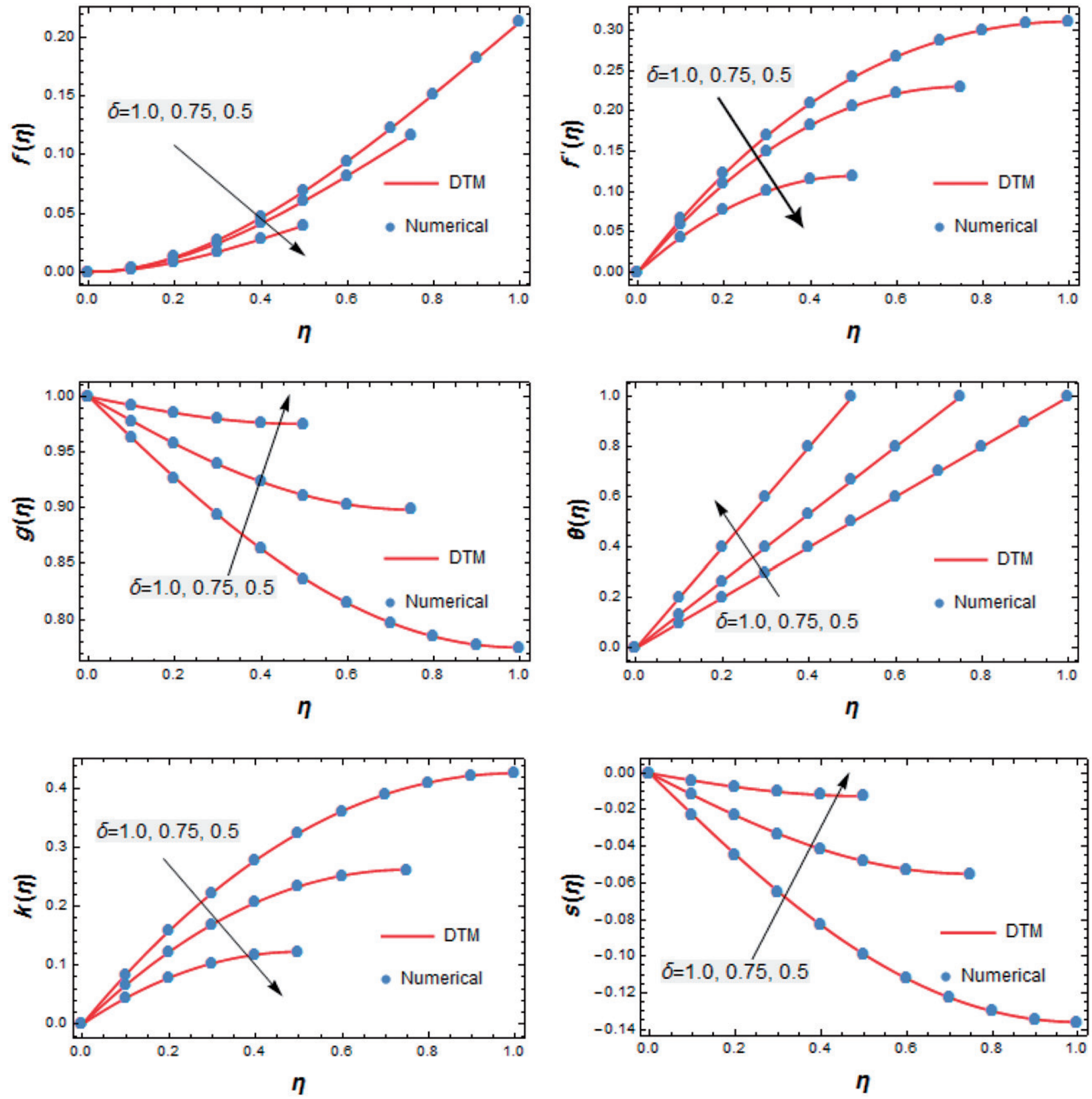


Figure 3. Comparison of DTM and Shooting for Cu-water nanofluid with $\phi = 0.04$ and $\delta = 0.5$, 0.75 and 1, respectively.

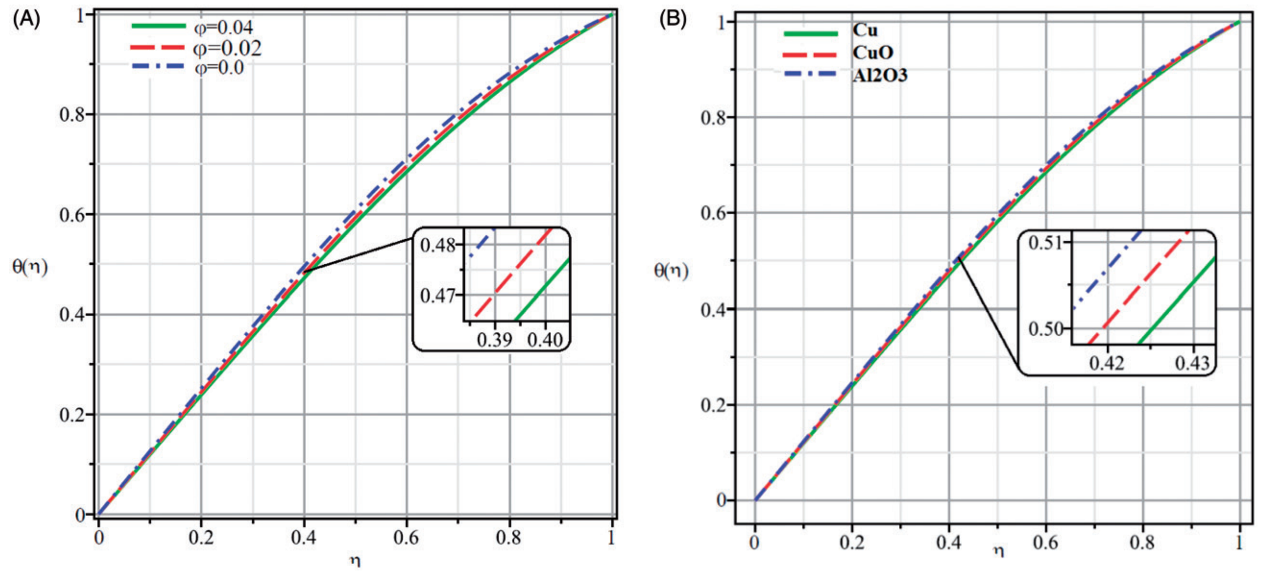


Figure 4. (A) Effect of nanoparticles volume fraction for Cu-water and (B) effect of nanoparticles material with $\phi = 0.04$ on temperature profile.

important parameters to evaluate how these parameters influence on this fluid. Effect of normalized thickness on velocity and temperature profiles by numerical and analytical solutions are shown in Figures 2 and 3. Increasing normalized thickness leads to increase in f, f' and decrease in g, θ. Effect of normalized thickness on k and s are similar to those of f' and g, respectively.

Figure 4 shows the effect of nanoparticles volume fraction for Cu-water and the effect of nanoparticles material on temperature profile. The sensitivity of thermal boundary layer thickness to volume fraction of nanoparticles is related to the increased thermal conductivity of the nanofluid. In fact, higher values of thermal conductivity are accompanied by higher values of thermal diffusivity. The high values of thermal diffusivity cause a drop in the temperature gradients and accordingly increase the boundary thickness. Also this figure shows that selecting aluminium oxide (Al_2O_3) as nanoparticle obtained more enhancement in temperature profile.

5. Conclusions

In this paper, three-dimensional nanofluid flow of condensation film on inclined rotating disk was solved via a sort of analytical methods (least square method (LSM) and differential transform method (DTM)) and numerical methods (the Runge-Kutta method of order 4 and Shooting method). Present analytical methods is a powerful approach for solving nonlinear differential equations such as this problem, also it can be observed that there is a good agreement between the present and numerical results. Results show that increasing nanofluid volume fraction leads to increase in temperature profile and the highest temperature was obtained for Al_2O_3-water in this case.

Acknowledgments

The authors gratefully acknowledge the financial support of Esfarayen University of Technology for the related research-plan defined in that university.

Disclosure statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work. There is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

References

Aziz, A. (2006). *Heat conduction with maple*. Philadelphia: RT Edwards.

Bhatti, M. M., & Zeeshan, A. (2017). Heat and mass transfer analysis on peristaltic flow of particle- fluid suspension with slip effects. *Journal of Mechanics in Medicine and Biology*, *17*(02), 1750028. Retrieved from https://doi.org/10.1142/S0219519417500282

Bhatti, M. M., Zeeshan, A., & Ellahi, R. (2016). Endoscope analysis on peristaltic blood flow of Sisko fluid with Titanium magneto-nanoparticles. *Computers in Biology and Medicine*, *78*, 29–41. Retrieved from https://doi.org/10.1016/j.compbiomed.2016.09.007

Bhatti, M. M., Zeeshan, A., & Ellahi, R. (2017a). Simultaneous effects of coagulation and variable magnetic field on peristaltically induced motion of Jeffrey nanofluid containing gyrotactic microorganism. *Microvascular Research*, *110*, 32–42. Retrieved from https://doi.org/10.1016/j.mvr.2016.11.007

Bhatti, M. M., Zeeshan, A., Ellahi, R., & Ijaz, N. (2017b). Heat and mass transfer of two-phase flow with Electric double layer effects induced due to peristaltic propulsion in the presence of transverse magnetic field. *Journal of Molecular Liquids*, *230*, 237–246. Retrieved from https://doi.org/10.1016/j.molliq.2017.01.033

Devakar, M., Ramesh, K., Chouhan, S., & Raje, A. (2017). Fully developed flow of non-Newtonian fluids in a straight uniform square duct through porous medium. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *23*, 66–74. Retrieved from https://doi.org/10.1016/j.jaubas.2016.04.001

Ellahi, R. (2013). The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe: Analytical solutions. *Applied Mathematical Modelling*, *37*(3), 1451–1467. Retrieved from https://doi.org/10.1016/j.apm.2012.04.004

Ezzat, M. A. (1994). State space approach to unsteady two-dimensional free convection flow through a porous medium. *Canadian Journal of Physics*, *72*(5-6), 311–317. Retrieved from https://doi.org/10.1139/p94-045

Ezzat, M. A. (2001). Free convection effects on perfectly conducting fluid. *International Journal of Engineering Science*, *39*(7), 799–819. Retrieved from https://doi.org/10.1016/S0020-7225(00)00059-8

Ezzat, M. A. (2011). Thermoelectric MHD with modified Fourier's law. *International Journal of Thermal Sciences*, *50*(4), 449–455. Retrieved from https://doi.org/10.1016/j.ijthermalsci.2010.11.005

Ezzat, M. A. (2012). State space approach to thermoelectric fluid with fractional order heat transfer. *Heat and Mass Transfer*, *48*(1), 71–82. Retrieved from https://doi.org/10.1007/s00231-011-0830-8

Ezzat, M. A., Abbas, I. A., El-Bary, A. A., & Ezzat, S. M. (2014a). Numerical study of the stokes' first problem for thermoelectric micropolar fluid with fractional derivative heat transfer. *Magnetohydrodynamics*, *50*(3), 263–277. Retrieved from http://doi.org/10.22364/mhd

Ezzat, M. A., El-Bary, A. A., & Ezzat, S. M. (2013). Stokes' first problem for a thermoelectric Newtonian fluid. *Meccanica*, *48*(5), 1161–1175. Retrieved from https://doi.org/10.1007/s11012-012-9658-7

Ezzat, M., Sabbah, A. S., El-Bary, A. A., & Ezzat, S. (2014b). Stokes' first problem for a thermoelectric fluid with fractional-order heat transfer. *Reports on Mathematical Physics*, *74*(2), 145–158. Retrieved from https://doi.org/10.1016/S0034-4877(15)60013-1

Ezzat, M. A., & Youssef, H. M. (2010). Stokes' first problem for an electro-conducting micropolar fluid with thermoelectric properties. *Canadian Journal of Physics*, *88*(1), 35–48. Retrieved from https://doi.org/10.1139/P09-100

Hatami, M., & Ganji, D. D. (2014a). Investigation of refrigeration efficiency for fully wet circular porous fins with variable sections by combined heat and mass transfer analysis. *International Journal of Refrigeration*, *40*, 140–151. Retrieved from https://doi.org/10.1016/j.ijrefrig.2013.11.002

Hatami, M., & Ganji, D. D. (2014b). Thermal and flow analysis of microchannel heat sink (MCHS) cooled by Cu-water nanofluid using porous media approach and least square method. *Energy Conversion and Management*, *78*, 347–358. Retrieved from https://doi.org/10.1016/j.enconman.2013.10.063

Hatami, M., Sheikholeslami, M., & Ganji, D. D. (2014). Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method. *Powder Technology*, 253, 769–779. Retrieved from https://doi.org/10.1016/j.powtec.2013.12.053

Hassani, M., Tabar, M. M., Nemati, H., Domairry, G., & Noori, F. (2011). An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Thermal Sciences*, *50*(11), 2256–2263. Retrieved from https://doi.org/10.1016/j.ijthermalsci.2011.05.015

Jensen, K. F., Einset, E. O., & Fotiadis, D. I. (1991). Flow phenomena in chemical vapor deposition of thin films. *Annual Review of Fluid Mechanics*, *23*(1), 197–232. Retrieved from https://doi.org/10.1146/annurev.fl.23.010191.001213

Khan, W., & Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, *53*(11), 2477–2483. Retrieved from https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032

Khanafer, K., Vafai, K., & Lightstone, M. (2003). Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International Journal of Heat and Mass Transfer*, *46*(19), 3639–3653. Retrieved from https://doi.org/10.1016/S0017-9310(03)00156-X

Mahanthesh, B., Gireesha, B. J., & Gorla, R. S. R. (2017). Unsteady three-dimensional MHD flow of a nano Eyring-Powell fluid past a convectively heated stretching sheet in the presence of thermal radiation, viscous dissipation and Joule heating. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *23*, 75–84. Retrieved from https://doi.org/10.1016/j.jaubas.2016.05.004

Mirgolbabaee, H., Ledari, S. T., & Ganji, D. D. (2017). Semi-analytical investigation on micropolar fluid flow and heat transfer in a permeable channel using AGM. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *24*, 213–222. Retrieved from https://doi.org/10.1016/j.jaubas.2017.01.002

Nawaz, M., Hayat, T., & Zeeshan, A. (2016). Melting heat transfer in an axisymmetric stagnation-point flow of the Jeffrey fluid. *Journal of Applied Mechanics and Technical Physics*, *57*(2), 308–316. Retrieved from https://doi.org/10.1134/S0021894416020140

Othman, M. I. A., & Ezzat, M. A. (2001). Electromagneto-hydrodynamic instability in a horizontal viscoelastic fluid layer with one relaxation time. *Acta mechanica*, *150*(1-2), 1–9. Retrieved from https://doi.org/10.1007/BF01178540

Rashidi, M. M., Abelman, S., & Mehr, N. F. (2013). Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid. *International Journal of Heat and Mass Transfer*, *62*, 515–525. Retrieved from https://doi.org/10.1016/j.ijheatmasstransfer.2013.03.004

Rashidi, M. M., & Dinarvand, S. (2009). Purely analytic approximate solutions for steady three-dimensional problem of condensation film on inclined rotating disk by homotopy analysis method. *Nonlinear Analysis: Real World Applications*, *10*(4), 2346–2356. Retrieved from https://doi.org/10.1016/j.nonrwa.2008.04.018

Saleh, H., Alali, E., & Ebaid, A. (2017). Medical applications for the flow of carbon-nanotubes suspended nanofluids in the presence of convective condition using Laplace transform. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *24*, 206–212. Retrieved from https://doi.org/10.1016/j.jaubas.2016.12.001

Sheikholeslami, M., Gorji-Bandpy, M., & Ganji, D. D. (2013a). Numerical investigation of MHD effects on Al2O3-water nanofluid flow and heat transfer in a semi-annulus enclosure using LBM. *Energy*, *60*, 501–510. Retrieved from https://doi.org/10.1016/j.energy.2013.07.070

Sheikholeslami, M., Hatami, M., & Ganji, D. D. (2013b). Analytical investigation of MHD nanofluid flow in a semi-porous channel. *Powder Technology*, *246*, 327–336.

Retrieved from https://doi.org/10.1016/j.powtec.2013.05.030

Sparrow, E. M., & Gregg, J. L. (1959). A theory of rotating condensation. *Journal of Heat Transfer*, *81*(2), 113–120.

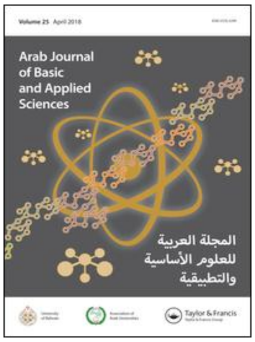
Srinivasacharya, D., & Shafeeurrahman, M. (2017). Hall and ion slip effects on mixed convection flow of nanofluid between two concentric cylinders. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, *24*, 223–231. Retrieved from https://doi.org/10.1016/j.jaubas.2017.03.002

Zeeshan, A., Hassan, M., Ellahi, R., & Nawaz, M. (2016). Shape effect of nanosize particles in unsteady mixed convection flow of nanofluid over disk with entropy generation. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, *231*(4), 871–879. https://doi.org/10.1177/0954408916646139

Zeeshan, A., & Majeed, A. (2016a). Heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction/injection and magnetic dipole effect. *Alexandria Engineering Journal*, *55*(3), 2171–2181. Retrieved from https://doi.org/10.1016/j.aej.2016.06.014

Zeeshan, A., & Majeed, A. (2016b). Effect of magnetic dipole on radiative non-Darcian mixed convective flow over a stretching sheet in porous medium. *Journal of Nanofluids*, *5*(4), 617–626. Retrieved from https://doi.org/10.1166/jon.2016.1237

Zhou, J. K. (1986). Differential transformation and its applications for electrical circuits (pp. 1279–1289). Wuhan: Huazhong University Press.



Taylor & Francis
Taylor & Francis Group

Arab Journal of Basic and Applied Sciences

ISSN: (Print) 2576-5299 (Online) Journal homepage: http://www.tandfonline.com/loi/tabs20

Some properties of binomial coefficients and their application to growth modelling

Vladimir L. Gavrikov

To cite this article: Vladimir L. Gavrikov (2018) Some properties of binomial coefficients and their application to growth modelling, Arab Journal of Basic and Applied Sciences, 25:1, 38-43, DOI: 10.1080/25765299.2018.1449346

To link to this article: https://doi.org/10.1080/25765299.2018.1449346

Published online: 09 May 2018.

ARAB JOURNAL OF BASIC AND APPLIED SCIENCES
2018, VOL. 25, NO. 1, 38–43
https://doi.org/10.1080/25765299.2018.1449346

University of Bahrain
Taylor & Francis
Taylor & Francis Group

ORIGINAL ARTICLE
OPEN ACCESS
Check for updates

Some properties of binomial coefficients and their application to growth modelling

Vladimir L. Gavrikov

Institute of Ecology and Geography, Siberian Federal University, Krasnoyarsk, Russian Federation

ABSTRACT

Some properties of diagonal binomial coefficients were studied in respect to frequency of their units' digits. An approach was formulated that led to the use of difference tables to predict if certain units' digits can appear in the values of binomial coefficients at quadratic terms of the binomial theorem. Frequency distributions of units' digits of binomial coefficients contain gaps (zero frequency) under most common numbering systems with supposed exclusion to systems with 2^n bases. In the work, an application of binomial coefficient arithmetic to model cell population dynamics was suggested. For a multicellular organism, the growth of number of cells was presented as a succession of binomial coefficients. It was inferred that the number of cells in a multicellular organism may obey power function laws.

ARTICLE HISTORY
Received 6 June 2017
Accepted 28 October 2017

KEYWORDS
Binomial coefficients; natural sequence; minor total; multicellular growth

1. Introduction

Since long ago binomial coefficients are known to appear in the binomial theorem of power decomposition of:

$$(1+x)^n = 1 + nx + C_n^2 \cdot x^2 + C_n^3 \cdot x^3 + \ldots + C_n^k \cdot x^k + \ldots, \quad (1)$$

where, x being a real variable, C_n^2, C_n^3, C_n^k, etc., are binomial coefficients. Because the first $k+1$ terms of the power decomposition is a polynomial of x at degree k the expressions C_n^2, C_n^3 are coefficients at quadratic, cubic terms, correspondingly. One of the most vivid representations of the coefficients is Pascal triangle (Figure 1) which is a triangular table where every nth row is the coefficients of the decomposition.

Arithmetical properties of binomial coefficients are well known and relate mostly to the divisibility by prime numbers and their degrees (Guo, 2013; Guo & Krattenthaler, 2014; Winberg, 2008) as well as to sums of the coefficients in one row, i.e. where n is a constant (Figure 1(a)). Besides subsets of the coefficients in horizontal rows, a theory has been developed which covers divisibility properties of central (or middle) binomial coefficients (Chen, 2016; Pomerance, 2015).

It has been shown (Gavrikov, 2017) that some subsets of the coefficients from diagonals of the Pascal triangle (Figure 1(b)) can possess other properties as well. Particularly, the frequency distributions of units' digits of binomial coefficients at quadratic terms (which are minor totals of natural sequence) may or may not have gaps dependently on numbering system considered. This property was strictly proven on the example of base-three and base-four numbering systems. In terms of modular arithmetic, the properties look as follows, l being a non-negative integer and S_l being the minor total of the first l terms of the natural sequence (the minor totals are numerically equivalent to binomial coefficients C_n^2 at quadratic terms). So, $S_{3l+0} \equiv 0 \pmod 3$, $S_{3l+1} \equiv 1 \pmod 3$, $S_{3l+2} \equiv 0 \pmod 3$ while $S_n \not\equiv 2 \pmod 3$ at any n. Then, $S_{4l+0} \equiv 0 \pmod 4$ at even l, $S_{4l+0} \equiv 2 \pmod 4$ at odd l, $S_{4l+1} \equiv 1 \pmod 4$ at even l, $S_{4l+1} \equiv 3 \pmod 4$ at odd l.

Gavrikov (2017) has also suggested how to prove presence or absence of gaps (omissions) in frequency distributions of units' digits of minor totals of natural sequence under numbering system with other bases. Empirically, the gaps are found in numbering systems with bases 5, 6, 7, 9 and 10 while there are no gaps in systems with bases 4, 8 and 16. For example, empirical frequency distributions of units' digits for base-seven and base-eight numbering systems are given in Figure 2.

The aim of the work was to explore some properties of diagonal binomial coefficients which are coefficients at quadratic and cubic terms of the binomial theorem. Units' digits of the diagonal binomial coefficients may have definite omissions (gaps in frequency distributions) depending on numbering system used. It would be therefore interesting to

CONTACT Vladimir L. Gavrikov vgavrikov@sfu-kras.ru Institute of Ecology and Geography, Siberian Federal University, 79 pr. Svobodnyi, Krasnoyarsk 660041, Russian Federation

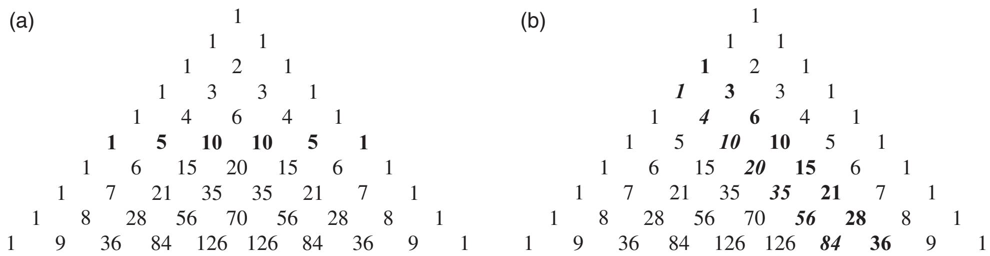


Figure 1. Pascal triangle. a – A horizontal row with constant n ($n = 5$), as an example, is highlighted in bold face; b – diagonals of the triangle are highlighted in which n is a variable. Minor totals of natural sequence (binomial coefficients at the quadratic term) are given in bold face. Binomial coefficients at the cubic term are given in bold italic face.

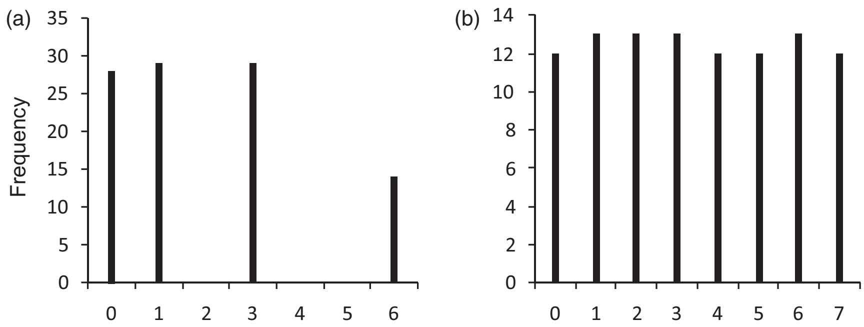


Figure 2. Empirical frequency distributions of units' digits in binomial coefficients at the quadratic term (Figure 1(b)). a – Base-seven numbering system; b – base-eight numbering system. Numbers at abscissa axis are digits of the corresponding numbering systems. Frequency is the number of the cases among first hundred of binomial coefficients values.

develop a tool which allows both proving the omission existence and predicting which omissions may be expected under a numbering system used. Another aim was to find if binomial coefficient arithmetic may be applied to modelling of cell population growth.

2. Generalization

The binomial coefficients at the quadratic term are minor totals of natural sequence (Figure 1(b)) and have been known since long ego. The minor totals themselves are a particular case of Faulhaber polynomials with unity exponent and are given by Faulhaber's formula (also called Bernoulli formula). The nth minor total of the sequence S_n is:

$$S_n = \sum_{\kappa=1}^{n} \kappa = \frac{n(n+1)}{2}. \quad (2)$$

The basic equation of the analysis looks like:

$$S_{Lk+i} = L \cdot m + j, \quad (3)$$

where, L is the base of a numbering system, i and j are units' digits in different representations of minor totals S_n, obviously $0 \leq i, j \leq (L-1)$. Values of k and m are non-negative integers, which is of core importance.

According to relation 2:

$$S_{Lk+i} = \frac{L \cdot L \cdot k^2 + L \cdot k(2i+1) + i(i+1)}{2}. \quad (4)$$

Then the generalizing relation linking m, k, i and j can be received from Equations (3) and (4) by expressing of m through all other parameters:

$$m = \frac{k(L \cdot k + (2i+1))}{2} + \quad (5)$$

$$+ \frac{\frac{i(i+1)}{2} - j}{L}. \quad (6)$$

The wholeness of m depends on evenness/oddness of term $k(L \cdot k + (2i+1))$ and on whether term $\frac{i(i+1)}{2} - j$ is divisible by L with or without a remainder. At given k and L, those combinations of i and j that ensure wholeness of m determine $S_{Lk+i} \equiv j \ (mod \ L)$.

Thus the generalizing relation 5–6 can be used to predict the units' digits in values of binomial coefficients at quadratic terms (S_n) at a given base of numbering system L.

Properties of term 5 depend on evenness/oddness of k and L. If L is odd then term 5 is always an integer ($k(L \cdot k + (2i+1)) \equiv 0 \ (mod \ 2)$). If L is even then at even k term 5 is an integer but at odd k the term 5 is a fraction: ($k(L \cdot k + (2i+1)) \equiv \frac{1}{2} (mod \ 2)$).

It is easy to see that $\frac{i(i+1)}{2} - j$ brings about a table (Table 1) that may called a "difference table" which contains all the possible differences between $\frac{i(i+1)}{2}$ and j. This table can be used to find the sought quantities of j.

Remark 2.1. The difference table may be used to predict not only units' digits in S_n but other digits as well. For example, considering $S_{L^2 k+i} = L^2 \cdot m + j$ one can study appearance of tens-and-units' digits of

Table 1. A fragment of difference table.

	(i(i+1))/2								
j	0	1	3	6	10	15	21	28	...
0	0	1	3	6	10	15	21	28	...
1	−1	0	2	5	9	14	20	27	...
2	−2	−1	1	4	8	13	19	26	...
3	−3	−2	0	3	7	12	18	25	...
4	−4	−3	−1	2	6	11	17	24	...
5	−5	−4	−2	1	5	10	16	23	...
6	−6	−5	−3	0	4	9	15	22	...
7	−7	−6	−4	−1	3	8	14	21	...
...	...	...	...	...	...	...	...	...	...

First eight values of (i(i+1))/2 and j are shown.

Table 2. A 7 × 7 difference table.

	(i(i+1))/2						
j	0	1	3	6	10	15	21
0	0	1	3	6	10	15	21
1	−1	0	2	5	9	14	20
2	−2	−1	1	4	8	13	19
3	−3	−2	0	3	7	12	18
4	−4	−3	−1	2	6	11	17
5	−5	−4	−2	1	5	10	16
6	−6	−5	−3	0	4	9	15

Values of differences divisible by 7 without a remainder are given on grey background. Values of j satisfying the condition "m is a non-negative integer" are given in bold face.

Table 3. A 8 × 8 difference table.

	(i(i+1))/2							
J	0	1	3	6	10	15	21	28
0	0	1	3	6	10	15	21	28
1	−1	0	2	5	9	14	20	27
2	−2	−1	1	4	8	13	19	26
3	−3	−2	0	3	7	12	18	25
4	−4	−3	−1	2	6	11	17	24
5	−5	−4	−2	1	5	10	16	23
6	−6	−5	−3	0	4	9	15	22
7	−7	−6	−4	−1	3	8	14	21

Values of differences divisible by 8 without a remainder or with a remainder 1/2 or −1/2 are given on grey background. Values of j satisfying the condition "m is a non-negative integer" are given in bold face.

Table 4. A 9 × 9 difference table.

		(i(I+1))/2								
j	j_T	0	1	3	6	10	15	21	28	36
0	0	0	1	3	6	10	15	21	28	36
1	1	−1	0	2	5	9	14	20	27	35
2	2	−2	−1	1	4	8	13	19	26	34
3	10	−3	−2	0	3	7	12	18	25	33
4	***11***	−4	−3	−1	2	6	11	17	24	32
5	***12***	−5	−4	−2	1	5	10	16	23	31
6	20	−6	−5	−3	0	4	9	15	22	30
7	***21***	−7	−6	−4	−1	3	8	14	21	29
8	***22***	−8	−7	−5	−2	2	7	13	20	28

j_T is a ternary representation of j numbers. Values of differences containing 9 are given on grey background. Tens-and-units values of j_T that do not satisfy the condition "m is a non-negative integer" are given in bold italic face.

base-L numbering system. An instance for ternary numbering system is given in the next section.

3. Propositions

In this section, examples of difference table usage are considered. These are cases of base-seven, base-eight and ternary numbering systems.

Proposition 3.1. In base-seven numbering system, $S_n \equiv j\ (mod\ 7)$ where $j \in \{0,1,3,6\}$ while $S_n \not\equiv \alpha\ (mod\ 7)$ where $\alpha \in \{2,4,5\}$ for any n.

Proof. Because 7 is an odd number, term 5 is an integer. Therefore combinations of i and j are sought that ensure the wholeness of term 6. In other words, the values of the difference table must be divisible by 7 without a remainder.

For base-seven numbering system, one should consider a 7 × 7 difference table (Table 2). As follows from the table, only values of $j \in \{0,1,3,6\}$ satisfy to the condition "m is a non-negative integer" and therefore $S_n \equiv j\ (mod\ 7)$ only from this set (Figure 2(a)).

Proposition 3.2. In base-eight numbering system, $S_n \equiv j\ (mod\ 8)$ where

$$j \in \{0,1,2,3,4,5,6,7\}.$$

Proof. Because 8 is an even number, the term 5 may be non-negative integer (for even k) and a fractional number (for odd k). Namely, for odd k $\frac{k(L\cdot k+(2i+1))}{2} \equiv \frac{1}{2}(mod\ 2)$. Therefore, the differences are sought that are divisible by 8 either without a remainder or with the remainder 1/2 or −1/2.

A 8 × 8 difference table should be considered (Table 3). As follows from the table, all the digits of the base-eight numbering system satisfy the condition "m is a non-negative integer", i.e. $S_n \equiv j\ (mod\ 8)$ for all the $j \in \{0,1,2,3,4,5,6,7\}$ (Figure 2(b)).

Remark 3.3. Not all even L will bring about that j covers all the digits of the numbering system. For example, for $L = 6$, $L = 10$ and others there will be gaps in the sets of units' digits. Empirically, only the cases $L = 2^c$ (c being a non-negative integer) lead to that j covers all the system digits.

Proposition 3.4. In ternary system, $S_n \not\equiv 11\ (mod\ 100)$ and $S_n \not\equiv 21\ (mod\ 100)$ for any n.

Proof. Converting the relations to decimal system one gets $S_n \not\equiv 4\ (mod\ 9)$ and $S_n \not\equiv 7\ (mod\ 9)$.

A 9 × 9 difference table should be considered in which difference values are sought that contain 9 (Table 4). As follows from the table, neither ternary 11 nor ternary 21 are present among values appearing as tens-and-units' digits in ternary representation of S_n.

The approach of difference tables will naturally work for other subsets of binomial coefficients. Consider the case of binomial coefficients at cubic term of the binomial theorem (Figure 1(b)). The basic equation to analyze is:

$$T_{Lk+i} = Lm + j, \quad (7)$$

where, T_{Lk+i} is a representation of the binomial coefficients at cubic term. Obviously, the values of the

Table 5. A 10 × 10 difference table.

	$(i(i+1)(i+2))(2\cdot 3)$									
j	0	1	4	10	20	35	56	84	120	165
0	0	1	4	10	20	35	56	84	120	165
1	−1	0	3	9	19	34	55	83	119	164
2	−2	−1	2	8	18	33	54	82	118	163
3	−3	−2	1	7	17	32	53	81	117	162
4	−4	−3	0	6	16	31	52	80	116	161
5	−5	−4	−1	5	15	30	51	79	115	160
6	−6	−5	−2	4	14	29	50	78	114	159
7	−7	−6	−3	3	13	28	49	77	113	158
8	−8	−7	−4	2	12	27	48	76	112	157
9	−9	−8	−5	1	11	26	47	75	111	156

Values of differences producing residues 0 or 1/2 are given on grey background.

coefficients may be obtained with the help of $\frac{(Lk+i)(Lk+(i+1))(Lk+(i+2))}{2\cdot 3}$.

Thus, the Equation (7) may be re-written as:

$$\frac{(Lk+i)(Lk+(i+1))(Lk+(i+2))}{2\cdot 3} = Lm + j, \tag{8}$$

from which the value of coefficient m may be expressed as:

$$m = \frac{k(L^2\cdot k^2 + Lk(3i+3) + (i+1)(i+2) + i(2i+3))}{2\cdot 3} + \tag{9}$$

$$+\frac{\frac{i(i+1)(i+2)}{2\cdot 3} - j}{L}. \tag{10}$$

As it has been shown above, the key point is to find such a combination of L, i and j that ensures that m is a non-negative integer. The term (9) produces residues which are multiples of 1/6: 0, 1/6, 2/6, 3/6, 4/6 and 5/6. Consequently, term (10) has to produce the same row of multiples of 1/6 to provide the wholeness of the sum. The expression $\frac{i(i+1)(i+2)}{2\cdot 3} - j$ is a difference table and the values of the table have to be checked for divisibility by L. Let's consider an example case of base-ten numbering system, i.e. $L = 10$.

Proposition 3.5. In base-ten numbering system, $T_n \not\equiv 2 \pmod{10}$, $T_n \not\equiv 3 \pmod{10}$, $T_n \not\equiv 7 \pmod{10}$ and $T_n \not\equiv 8 \pmod{10}$ for any n.

Proof. A 10 × 10 difference table should be considered in which difference values are sought that while divided by 10 produce residues from 0, 1/6, 2/6, 3/6, 4/6 or 5/6 (Table 5). Obviously, a division by 10 can produce only the residues of 0 and 3/6 (i.e. 1/2). As follows from the table, the rows corresponding to j equal to 2, 3, 7 and 8 contain no values that produce the residues 0 or 1/2. Therefore, the relations given in the proposition are true. Also, empirical distribution of units' digits (Figure 3) supports the inference.

4. Binomial coefficients and multicellular growth

There are some general similarities between binomial coefficients and numbers of cells in an organism.

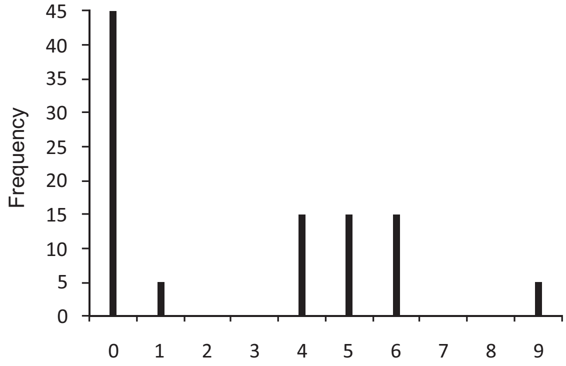


Figure 3. Empirical frequency distribution of units' digits in binomial coefficients at the cubic term at base-ten numbering system. Numbers at abscissa axis are digits of the corresponding numbering system. Frequency is the number of the cases among first hundred of binomial coefficients values.

Both are non-negative integers; binomial coefficients are generated by a summation algorithm as well as new cells are added to the existing cells giving rise to growth. The binomial coefficients arithmetic could be therefore considered in terms of biological growth modelling.

A classical example of non-negative integer modelling is provided by a consideration of population growth of cells each of which doubles in equal time intervals producing the next cell generation. Such a dynamics gives rise to sequence of the cell numbers like 1, 2, 4, 8, …, 2^β, β being the number of the cell generation while value 2 in this particular case being the reproduction factor. In other words, this is an example of how local divisions of cell lead to an *exponential law* of cell population growth.

Within a multicellular organism, however, the cell population growth obeys much more complicated rules. On the example of higher plant organisms, the following rules (kinds of cells) may be identified. First, there are the so-called initial cells that preserve the ability to divide in the course of the entire life span of the organism. Second, the cells – immediate descendants of the initials – can divide for some time but sooner or later transform to the third kind of cells. The third kind are remote descendants that are differentiated cells that may be dead or alive but no longer divide. Because the amount of initial cells compared to other kinds is low, a modelling may "imply" their existence but not to take into account explicitly.

Let's M be the number of differentiated cells and m be the number of dividing cells at the moment (generation number) 0. The next moment (generation) the number of dividing cells is multiplied by 2 – each of them divides in two. A share of the new cells goes to the pool of differentiated cells and another share remains to be dividing. In order to get a law of the whole population growth, it is necessary to suppose how the number of dividing cells alters from generation to generation. Why their number can change is a question of a separate sort and not considered here in detail. One can note, however, that the number of newly differentiated cells may influence the number of the new generation of dividing cells.

Suppose, first, that the number of dividing cells grows from generation to generation linearly, namely, by one cell a generation. Then the whole dynamics of population of N cells in generations from 0 to q may look as in expressions 11–15. In Expression 12 (generation 1), for example, the term $2m-(m+1)$ denotes a new portion of differentiated cells while $(m+1)$ is the new amount of dividing cells which is by one cell bigger as in generation 0:

$$M+m \qquad N_0 \tag{11}$$

$$M+2m-(m+1)+(m+1) \qquad N_1 \tag{12}$$

$$M+2m-(m+1)+2(m+1)-(m+2)+(m+2) \qquad N_2 \tag{13}$$

$$\begin{aligned} &M+2m-(m+1)+2(m+1)-(m+2)+2(m+2)- \\ &-(m+3)+(m+3) \qquad N_3 \\ &\ldots \end{aligned} \tag{14}$$

$$\begin{aligned} &M+2m+2(m+1)+2(m+2)+\ldots+2(m+q-1)- \\ &-[(m+1)+(m+2)+\ldots+(m+q-1)] \qquad N_q \end{aligned} \tag{15}$$

In Expression 15, collecting terms and taking into account that $1+2+3+\ \ldots\ +q-1$ is $\frac{(q-1)q}{2}$ give N_q, the size of the cell population at generation q in the form:

$$N_q = M+m+m\cdot q+\frac{(q-1)q}{2}, \tag{16}$$

which is an equivalent to coefficients of the binomial theorem (1) truncated to the quadratic term. In other words, the idealized multicellular organism grows as minor totals of the natural sequence. A simple corollary of the dynamics is that the growth obeys a *power law*, in this particular case it is a quadratic power function. It is easy to show that if the number of dividing cells stays constant then the entire organism grows linearly. If the number of dividing cells falls linearly, then the entire organism undergoes a decay of growth which obeys a power law as well.

The number of dividing cells may alter in a nonlinear manner. Suppose then that their number grows as the minor totals of the natural sequence, which is the same as binomial coefficients at the quadratic term. As seen from Pascal triangle (Figure 1(b)), one can expect that the total growth should obey the pattern of binomial coefficients at the cubic term. In fact, relations between binomial coefficients at the quadratic and cubic terms, S_n and T_n, correspondingly, are rather transparent. The summation of S_n naturally gives T_n:

$$\begin{aligned} T_n &= \sum_{\kappa=1}^{n} S_\kappa = \sum_{\kappa=1}^{n} \frac{\kappa(\kappa+1)}{2} = \frac{1}{2}\sum_{\kappa=1}^{n}\kappa^2+\frac{1}{2}\sum_{\kappa=1}^{n}\kappa = \\ &= \frac{n}{12}(n+1)(2n+1)+\frac{n}{4}(n+1) = \frac{n(n+1)(n+2)}{2\ \cdot\ 3} \end{aligned} \tag{17}$$

$$M+\frac{m(m+1)}{2} \qquad N_0 \tag{18}$$

$$M+m(m+1)-\frac{(m+1)(m+2)}{2}+\frac{(m+1)(m+2)}{2} \qquad N_1 \tag{19}$$

$$\begin{aligned} &M+m(m+1)-\frac{(m+1)(m+2)}{2}+(m+1)(m+2)- \\ &-\frac{(m+2)(m+3)}{2}+\frac{(m+2)(m+3)}{2} \qquad N \end{aligned} \tag{20}$$

$$\begin{aligned} &M+m(m+1)-\frac{(m+1)(m+2)}{2}+(m+1)(m+2)- \\ &-\frac{(m+2)(m+3)}{2}+(m+2)(m+3)- \\ &-\frac{(m+3)(m+4)}{2}+\frac{(m+3)(m+4)}{2} \qquad N_3 \\ &\ldots \end{aligned} \tag{21}$$

$$\begin{aligned} &M+m(m+1)+(m+1)(m+2)+(m+2)(m+3)+\ldots \\ &+(m+q-1)(m+q)+\frac{(m+q)(m+q+1)}{2}- \\ &-\left(\frac{(m+1)(m+2)}{2}+\frac{(m+2)(m+3)}{2}+\ldots\right. \\ &\left.+\frac{(m+q-1)(m+q)}{2}+\frac{(m+q)(m+q+1)}{2}\right) \qquad N_q \end{aligned} \tag{22}$$

Expressions 18–22 show the whole cell population dynamics in detail implying that the cell population consists of M differentiated and $\frac{m(m+1)}{2}$ dividing cells at the generation 0. Analogically to Expression 12, in 19, $m(m+1)-\frac{(m+1)(m+2)}{2}$ is the new differentiated cell while $\frac{(m+1)(m+2)}{2}$ is the new number of dividing cells.

Collecting terms in Expression 22 one gets:

$$\begin{aligned} N_q &= M+m(m+1)+\frac{(m+1)(m+2)}{2} \\ &+\frac{(m+2)(m+3)}{2}+\ldots+\frac{(m+q-1)(m+q)}{2}, \end{aligned} \tag{23}$$

which is with the help of Equations (2) and (17) transferred to:

$$\begin{aligned} N_q &= M+S_m+S_m+S_{(m+1)}+S_{(m+2)}+\ldots+S_{m+q-1} = \\ &= M+S_m+T_{m+q-1}-T_{m-1}. \end{aligned} \tag{24}$$

In Equation (24), the term $M+S_m-T_{m-1}$ is a mere constant depending on initial conditions (M and m). The term T_{m+q-1} gives dynamics of the number of cells in the idealized multicellular organism. Obviously, T_{m+q-1} is a polynomial of the third order.

An important implication of the cell population behaviour modelled with diagonal binomial coefficient arithmetic is that it is a power law that governs the growth of a multicellular organism, naturally, in terms of cell number. In mathematical modelling of biological objects, the use of polynomial forms at approximating of various biological data has been called into question. While providing high approximation accuracy, the polynomial forms often lack profound justifications and interpretability of parameters. The approach presented in the above analysis may help to fill the gap in polynomial usage in biological growth modelling.

To give a summary, an approach of difference tables was suggested in the work which provides a tool to study some properties of diagonal binomial coefficients. The properties are presence or absence of certain units' digits in the coefficients, that is, gaps in frequency distributions of the units' digits. The difference tables help to formulate mathematical proves for any numbering system. Hypothetically, the gaps will be found under all numbering systems which do not have bases of 2^c (c being a non-negative integer). It has been also suggested to use binomial coefficients arithmetic to model multicellular growth in term of number of cells. The use of the arithmetic implies application of power laws to model the growth.

Acknowledgements

The author is grateful to Dr. Vladimir Zahvataev who provided fruitful discussions in the course of the manuscript preparation. The comments of three anonymous reviewers are highly appreciated that helped to improve the manuscript.

Disclosure statement

No potential conflict of interest was reported by the author.

References

Chen, H. (2016). Interesting series associated with central binomial coefficients, Catalan numbers and harmonic numbers. *Journal of Integer Sequences*, *19*, 3.

Gavrikov, V. L. (2017). Some features of natural sequence totals in various numeral systems. arXiv preprint. Retrieved May 6, 2017, from http://arxiv.org/abs/1703.10475

Guo, V. J. (2013). Proof of two divisibility properties of binomial coefficients conjectured by Z.-W. Sun. arXiv preprint. Retrieved May 6, 2017, from https://arxiv.org/abs/1312.7548

Guo, V. J., & Krattenthaler, C. (2014). Some divisibility properties of binomial and q-binomial coefficients. *Journal of Number Theory*, *135*, 167–184. https://doi.org/10.1016/j.jnt.2013.08.012

Pomerance, C. (2015). Divisors of the middle binomial coefficient. *The American Mathematical Monthly*, *122*, 636–644. doi:10.4169/amer.math.monthly.122.7.636

Winberg, E. B. (2008). Amazing arithmetical properties of binomial coefficients. *Matematicheskoe prosveschenie*, *12*, 33–42 (in Russian).